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Professor A. J. Sutton Pippard, M.B.E., D.Sc., M.I.C.E., Chairman of the Division, in the Chair.

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Authors.

Structural Paper No. 31

"Buoyant Foundations in Soft Clay for Oil-Refinery Structures at Grangemouth"

by

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SYNOPSIS

In the design of the foundations for heavy load concentrations, maximum use was made of earth buoyancy to reduce the net soil bearing-pressure.

Each foundation consists of a hollow reinforced-concrete box below ground level. Longitudinal and transverse walls divide the foundation into a series of 10-foot-square cells. The walls were built to the full height above ground and sunk to the required levels as monoliths. The cell bottoms were laid during and immediately after the final stages of sinking. Subsequently, structural decks were formed over the cells.

Reinforced-concrete surface-rafts were used for founding light load concentrations.

It was anticipated that the settlements of the surface rafts would be greater than those of the buoyant foundations and allowance was made for this in the initial foundation levels. Records of settlements are being kept, and details for the first four buoyant foundations and the first surface raft to be built are set out in the Paper.

Introduction

The original Grangemouth refinery of Scottish Oils Ltd was sited on the bank of the River Forth immediately east of the Port of Grangemouth. When this site was chosen it was known that the soil consisted of a considerable but unexplored depth of soft clay with a somewhat firmer stony layer at about 30 feet below the surface. The plant was carried on reinforced-concrete rafts which were laid practically on the surface of the ground because the site was below the highest recorded tide level. The design was based on a maximum soil bearing-pressure of 0·5 ton per square foot although the average pressure did not exceed 0·42 ton per square foot. Since the rafts were built at ground level these pressures may be regarded as net.

About a year after construction, some slight settlements were observed at the more heavily-loaded points, and by 1948 up to 12 inches of settlement had occurred. The uneven loading of the boiler house raft had resulted in a tilt which brought the chimney 2 feet out of plumb in a height of 110 feet.

In 1930, 1937, and 1939 additional plant and boilers were installed on foundations consisting of bulb-ended cast-in-situ piles, the toes of which were driven into the stony layer existing at a depth of about 30 feet, the load per pile being about 15 tons. Some small settlement of this additional plant, with consequent slight distortion of the superstructure, has occurred.

The throughput capacity of the original refinery had remained unchanged from when it was opened in 1924; but in 1946, as part of the scheme for extension of home refining activities, the parent company, Anglo-Iranian Oil Co. Ltd, decided to increase the throughput capacity to five times the previous figure. This involved the construction of several heavy refining units together with associated treatment plants, tankage and services, the latter including a new power station of 20,000 kilovolt-amperes capacity. Each of the refining units consists of a group of main elements linked together by pipework. The characteristic main elements are furnaces for heating the oil, large reaction vessels, fractionating towers, banks of heat-exchangers, pump-houses, and chimney-stacks.

Flat land suitable in area and position for such an extension was available immediately east of the existing refinery and, although its soil bearing-capacity was low, the site was chosen as being the best available from the operating and commercial points of view.

Foundations for the New Works

The expansion programme demanded foundations capable of carrying a wide range of load concentrations, many of which were much heavier than had been necessary for the earlier works. The foundations were

divided into two categories, the heavier load concentrations being carried on buoyancy rafts and the lighter ones on surface rafts. The Paper deals mainly with the buoyancy rafts, the surface rafts being referred to only in connexion with the choice of types of foundation and with settlements.

It is established practice in foundation design to base calculations on the net soil bearing-pressure, which is obtained by deducting the weight of soil displaced by the foundation from the total load to be supported. In cases where the net bearing pressure cannot be reduced to the safe value for the site by spread surface-foundations, relief of pressure may be obtained by taking advantage of earth buoyancy. A buoyant foundation may be defined as one in which the net soil bearing-pressure is reduced to a practicable value by the use of hollow construction below ground level, so that the combined weight of superstructure and foundation equals the weight of the soil displaced by the foundation plus the soil bearing-capacity at the founding level.

At Grangemouth, buoyancy was obtained by use of reinforced-concrete cellular rafts. Monolith construction was adopted to obviate the problems which would have arisen in the large open excavations necessary for in-situ construction. Monoliths in various stages of sinking are shown in *Figs 1* (facing p. 308). Some settlement of both the surface and the buoyant foundations was anticipated, and allowance was made in the initial founding levels. On completion of the foundation works, arrangements were made for periodical recording of levels at a series of chosen points.

Soil Investigation

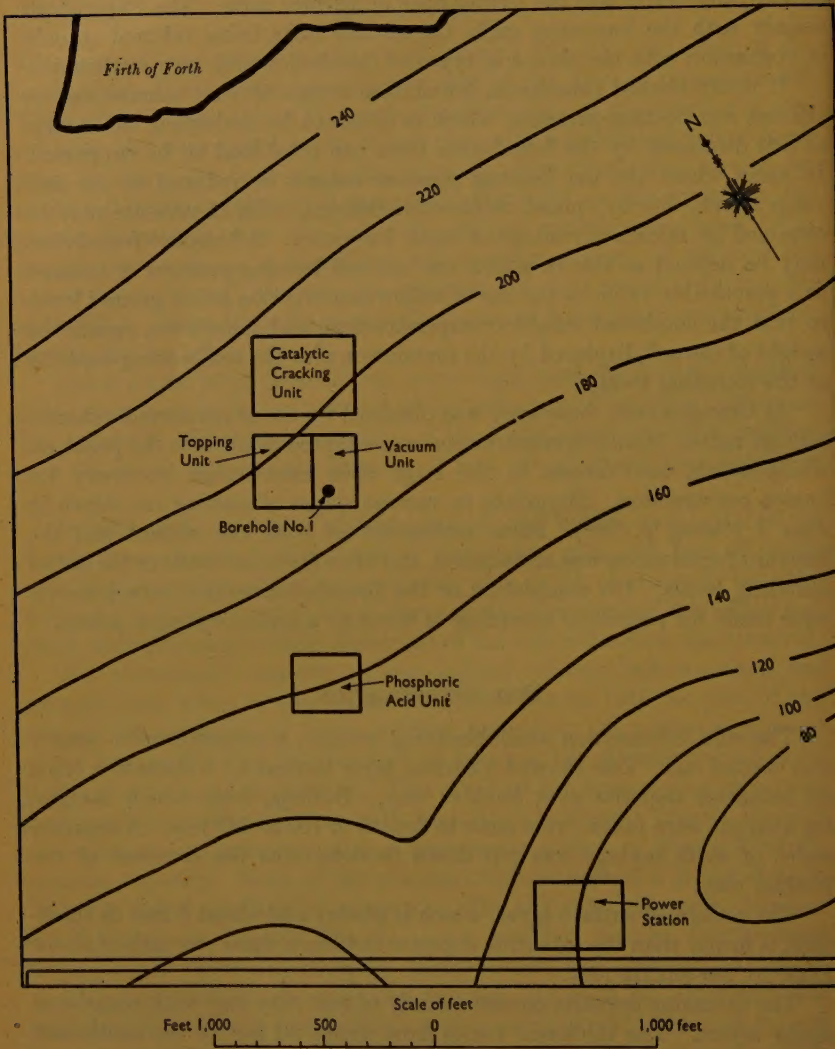
The soils information available being meagre, a comprehensive survey was carried out. This showed a surface layer formed by reclamation lying on estuarine deposits over boulder clay. Borings, from which samples for analysis were taken, were sunk to depths of 100 to 145 feet. A separate series of wash borings was put down to determine the contours of the boulder clay.

The reclaimed surface layer, which is of clay and about 5 feet in thickness, is firmer than the estuarine deposits and has a shear strength of about 1,000 lb. per square foot.

The estuarine deposits consist mainly of soft silty clay with occasional sandy layers. The thickness varies from about 80 feet in the south-east to more than 240 feet in the north-west corner of the site. Approximate contours of depths below ground level to the top of the boulder clay are shown in *Fig. 2*. The positions of the power station and refinery units, and that of borehole No. 1, have been superimposed on the contours. The soil properties of samples taken from this borehole are shown in *Figs 3*.

The soil properties shown in *Figs 3* are considered to be fairly typical of the whole site. Generally, the liquid limit is only a little higher than

Fig. 2

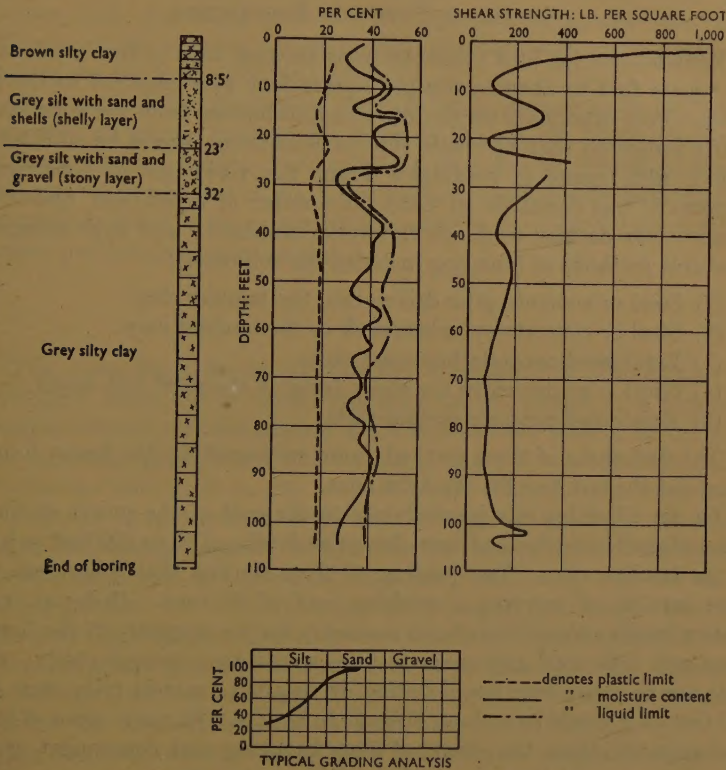


CONTOURS SHOWING DEPTH OF TOP OF BOULDER CLAY BELOW GROUND LEVEL

Note: Ground level is approximately +11·50 O.D.

the moisture content, which has an average value of about 35 per cent. The estimated shear strength ranges from 100 to 400 lb. per square foot with an average value of 250 lb. per square foot. To obtain undisturbed samples of the soils presented considerable difficulties, so a

Figs 3



TYPICAL SOIL PROPERTIES
(Borehole No. 1)

series of tests was made on the site with the vane instrument, newly-developed at that time. At depths of more than about 45 feet these tests showed greater shear strength, the increase being more marked with increasing depth ; at 100 feet the vane gave a reading of about 800 lb. per square foot. A more detailed account has been given by Dr A. W. Skempton.[1]

Immediately below the reclaimed surface layer there is a permeable layer containing shells. The moisture content is much higher than at other levels.

A fairly firm stony layer varying from 2 to 10 feet in thickness is found at levels between 23 and 33 feet. The matrix between the stones is comparatively coarse and the moisture content low.

[1] " Vane Tests in the Alluvial Plain of the River Forth, near Grangemouth." *Géotechnique*, vol. 1, No. 2, p. 111 (Dec. 1948).

Choice of Types of Foundation

Foundations for load concentrations ranging from a few hundred lb. per square foot to about a ton per square foot were required in the new works. Past experience on the site and information from the soils survey clearly indicated that no single type of foundation would be suitable for such a wide range of loadings. From the viewpoint of construction economy it was desirable to limit the number of types used, and it was consequently decided to divide the works into heavy and light categories. Available methods of founding included the following :—

(1) Steel or concrete piles driven into the boulder clay.
(2) Steel or concrete cylinders sunk to the boulder clay.
(3) Reinforced-concrete buoyancy rafts.
(4) Short piles driven to the stony layer at about 30 feet depth.
(5) Reinforced-concrete surface-rafts.

The first three of these methods were envisaged for the heavy foundations and the last two for the light ones.

On an adjoining site immediately to the east of the power station, a series of steel box-piles had been driven to depths of 90 to 100 feet to found in the boulder clay. Test-loading of these showed that individual piles were capable of carrying a working load of 60 tons. However, much greater lengths would have been necessary for the majority of the foundations and both steel and concrete piles would have proved costly. Steel piles can be easily lengthened during driving but concrete piles more than 100 feet long would have been difficult to handle. In many cases of heavy load-concentrations the effects of close grouping and consequent ground disturbance would have required careful consideration.

Steel or concrete cylinders were not considered, because they would have involved slow construction and high cost.

Reinforced-concrete buoyancy rafts could be designed, for the heaviest loadings, irrespective of the thickness of soft strata. Preliminary estimates indicated that these would be less costly than piled foundations. With the exception of timber for form-work, the materials required for their construction were, early in 1949, more readily available than steel piles. By maximum standardization of detail and the use of pre-cast concrete where possible, it was anticipated that the quantity of timber required for form-work could be kept to the minimum and that available supplies would meet the needs of the job. It was therefore decided that buoyancy rafts should be used throughout the works for founding all the heavy-category loadings.

Short piles founded in the stony layer at a depth of about 30 feet and reinforced-concrete surface-rafts had been used in the original refinery but, as previously mentioned, neither had been entirely successful. It was considered, however, that by limiting the net bearing-pressure to 500 lb. per square foot, instead of 0·5 ton per square foot as previously used, large

settlements would be avoided. Short piles could be relied on for about 15 tons each, but these possessed no technical advantage over surface rafts and would have been more costly. Consequently, surface rafts were adopted for the lightly-loaded structures.

As a result of the division of the foundation works into two categories, buoyancy rafts were used for the main building of the power station, the furnaces, reaction vessels, fractionating towers, banks of heat-exchangers, and larger pump-houses in the refinery areas; surface rafts were employed for ancillary items in the power station, and for the control-houses, switch-houses, and smaller pump-houses.

Design

The Consulting Engineers for the Grangemouth foundations had previously made use of earth buoyancy in the design of a reinforced-concrete cellular raft foundation for a large power station in India. In this case some positive net soil bearing-pressure was permissible and the foundations were built in situ without undue difficulties in retaining the sides or risk of upheaval of the bottom of large excavations. At Grangemouth the bearing capacity of the soils was so low that it was essential to make maximum use of earth buoyancy; that is to say, the net soil bearing-pressure was reduced to zero by arranging the foundation dimensions so as to provide a balance between weight of displaced soil and weight of superstructure plus foundation. Careful positioning of the foundations beneath the loads was necessary to ensure practically uniform net bearing-pressure, for otherwise there would have been risk of tilting.

The monolith construction adopted ensured some weight on the bottom of the excavations at all times, thus reducing the tendency to upheaval of the bottom owing to removal of earth loading. The cell bottoms were plugged with plain concrete as the monoliths were reaching the final levels, so that the soil at the founding level was exposed to the action of the weather for the shortest possible time. The sides of the monoliths retained the surrounding ground, rendering side slopes to the excavations unnecessary and thus reducing soil disturbance outside the monoliths to the minimum.

Before the detail designs were commenced, a trial shaft, 8 feet in diameter and 21 feet deep, lined with pre-cast concrete pipes, was sunk with the aim of amplifying the information obtained from the soils survey. Sinking was done in 3- to 4-foot stages by grabbing below the bottom of the pipes, their downward movement being assisted by kentledge. Inflow of water occurred when sinking through the permeable layer at a depth of about 7 feet, and pumping was necessary until more pipes were placed and had formed a seal. During the remainder of the work, pumping was not required although the bottom was wet at all times.

The difficulties involved in accurately assessing the amount and probable variation of loading made it impossible to ensure perfectly uniform

soil bearing-pressure. However, with the types of structure for which the foundations were designed, load variation was comparatively small. In the power station, the load was mainly the dead weight of structure and plant, and in the refinery the dead weight of plant and liquid loading, so that the proportion of incidental loading was small.

Normally, the operating load condition was used as a basis for design, a check being applied for the effect of wind. A further check was carried out for erection loading conditions with and without the effect of wind. During the erection of the superstructure, the net effective pressure was usually negative and in a material having little shear strength the foundation might have tended to rise. Though small, however, the shear value added to the friction between the outside faces of the foundations and the soils in contact with them was sufficient to counter the tendency to uplift and tilting.

The vessels, towers, and pipework systems were, in the main, test-loaded with water before the plant was put into operation. The load from these tests was usually somewhat greater than the operating load and owing to eccentricity the net bearing-pressure was not quite uniform. The tests took only a few days to carry out and some positive net bearing-pressure was permitted. Owing to the short period of application of the test loads the effect of wind was not considered. In a few cases the full test-loading was almost double the normal operating load and arrangements were made for the testing to be done in stages to avoid large eccentricities of loading and unduly high net bearing-pressures.

In all but the smallest monoliths the wall construction was checked for stresses likely to arise during sinking of the monoliths. Two cases were considered :—

(1) In which the monoliths were assumed to be supported by friction between the outer walls and the surrounding soil.
(2) In which the monoliths were assumed to be supported by the soil under the walls of a few cells in the centre.

It is probable that in practice neither of these conditions was fully realized, even in the largest monoliths.

From the viewpoints of both design and construction a rectangular plan shape was desirable, and for this reason cantilevers incorporated with the structural decks were frequently employed to support outlying items. In a few foundations the size was, to some extent, restricted by the proximity of underground pipework. Such a case was monolith No. 6 (see *Figs 4*), where thirty cells in a six-by-five arrangement would have been preferable to the six-by-four adopted. Sufficient buoyancy was, however, obtained by increase in depth. Where the plant arrangement was not suitable, departure from the rectangular shape was necessary. Several such cases occurred in the topping and vacuum units, and these are shown in *Figs 4*. Cellular foundation No. 7 was built in situ after completion

Fig. 1 (a)

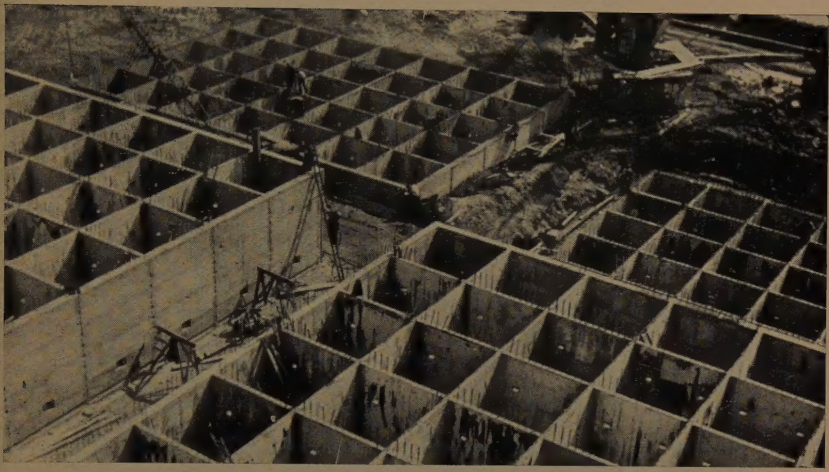


Fig. 1 (b)

POWER-STATION MONOLITHS DURING SINKING PERIOD

Fig. 5

COMPLETED STRUCTURAL DECK, MONOLITH NO. 5

Fig. 6 (a)

Pre-cast Wall Panels during Erection

Fig. 6 (b)

Pre-cast Wall Panels during Erection

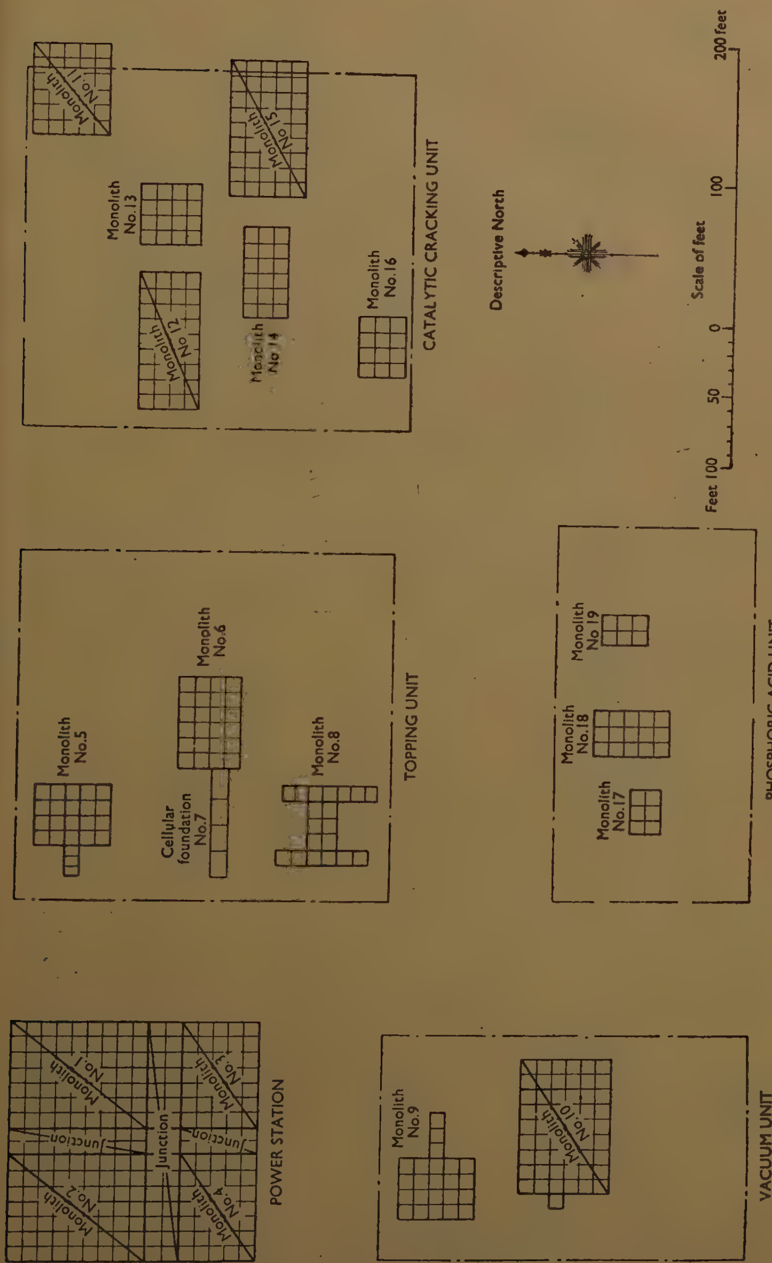
Fig. 7

DETAILS OF PRE-CAST WALL PANELS

Fig. 8

OPEN EXCAVATION AND CONCRETE BEARING PADS BEFORE ERECTION OF MONOLITH No. 6

Fig. 4



General Arrangements of Foundations

(The units are not shown in their true relative positions)

of monolith No. 6, but in all other cases the projecting portions were integral with the monolith construction.

The power-station foundation, about 170 feet square in plan, was considered too large for construction as one unit. Sinking stresses might have been high and control of the excavation difficult. Consequently, this foundation was built as four separate monoliths, arranged with north-to-south and east-to-west alleyways about 20 feet wide between them. After sinking, these monoliths were joined throughout their depths by in-situ cellular construction to form a single foundation.

Throughout the design, the aim was to produce light-weight foundation structures so that the superstructure-loading supported per ton of earth excavated was the maximum. In practice it was found that roughly 3 tons of displaced earth provided sufficient buoyancy to carry 2 tons of superstructure. More accurate figures are given in Table 1.

TABLE 1.—CHARACTERISTICS OF BUOYANCY FOUNDATIONS

Unit	Monolith number	Number of cells	Overall depth of foundation: ft. in.	Weight of foundation: tons	Weight of displaced earth: tons	Weight of foundation per unit vol.: lb./cu. ft.
Power station	1, 2, 3 & 4 Joined by cells of similar construction.	240	15 6	8,150	23,100	40·6
Topping unit	5	23	16 0	840	2,000	48·4
	6	24	20 3	970	2,980	37·4
	7	4	12 6	260	570	52·5
	8	19	16 3	820	1,810	52·1
Vacuum unit	9	23	17 3	880	2,370	42·7
	10	55	13 0	1,700	4,300	45·5
Catalytic Cracking unit	11	30	18 0	1,130	3,250	40·0
	12	36	20 0	1,400	4,300	37·4
	13	16	20 0	670	1,990	38·7
	14	18	19 0	730	2,060	40·8
	15	45	19 0	1,640	5,080	37·1
	16	12	16 0	460	1,190	44·5
Phosphoric Acid unit	17	6	17 0	270	650	47·7
	18	15	21 0	660	1,920	39·5
	19	6	16 3	260	620	48·2

The walls forming each monolith were erected on a prepared surface above ground. The internal walls were 6-inch-thick pre-cast reinforced-concrete panels joined at the intersections by in-situ concrete. Continuity

of the horizontal reinforcement was obtained by full-strength welding of the bar ends which projected from the sides of the pre-cast panels. The 12-inch-thick external walls were of in-situ construction and were poured after erection of the internal walls.

The cell bottoms were designed for construction in two stages, first a plain concrete plug and, secondly, a reinforced-concrete seal. During the final stages of monolith sinking the plugs were employed to assist in arresting the downward movement of the monoliths and later formed a dry bottom on which the seals were laid. The seals were cross-reinforced and continuity of reinforcement in adjacent seals was obtained by splice-bars inserted through holes cored in the pre-cast walls during manufacture.

The monoliths were capped with structural decks, consisting of a 12-inch thickness of in-situ reinforced concrete laid on top of 3-inch-thick pre-cast pre-stressed concrete planks. These planks, spanning between cell walls, served as formwork for the in-situ concrete, thus rendering unnecessary the use of large quantities of timber or other shuttering which would have been difficult to remove from inside the cells. The top surface of the pre-cast planks was corrugated to assist in bonding them to the in-situ concrete, the design having provided for their combined action in resisting the stresses. The structural decks were stiffened as necessary by spreader beams and plinths to carry local loading from brickwork, structural frameworks, and plant. *Fig. 5* * shows the completed deck of monolith No. 5.

The walls, bottoms, and structural decks forming the foundations were designed to resist (in combination) the main raft stresses, in addition to the stresses set up by local loading.

In the refinery-plant foundations it was necessary to take precautions against the accumulation of explosive gases in the cells. With this end in view, airtight access manholes were used in the structural decks, these being mounted on upstanding kerbs so that any oil which might be spilled would not leak into the cells.

Construction

Programme

To prevent risk of flooding, all areas to be developed were raised about 2 feet to bring the ground level up to + 13·75 O.D., which is the highest recorded tide-level at Grangemouth. The material used was spent oil-shale from the Oil Company's Scottish workings. This packed down to form a surface over which construction plant was able to move freely throughout the work.

The power station and topping unit were required to be in operation by the end of 1950, with other refinery units following later. Since the power station involved the greater amount of work in erection of superstructure and plant, work on its foundations was started as early as

* *Figs 5–8* inclusive are all photographs and appear between pp. 308 and 309.

possible, that is, in April 1949, the foundations for the topping unit being started 3 months later. The remaining units were brought into the programme subsequently; work on the phosphoric acid unit started in August 1949, on the catalytic cracking unit in June 1950, and on the vacuum unit in January 1951. Other works, not described in this Paper, but which were incorporated in the programme, included a series of surface-raft foundations in the units mentioned and various treatment plants and tankage. This programme was eminently suitable for stage-by-stage construction and the Contractors were able to keep a steady labour force on the foundations until the work was all but completed.

Internal Pre-cast Walls

Details of the pre-cast wall panels are shown in *Fig. 7*. The panels were cast lying flat, in stacks of about ten, separated by layers of building paper, thus reducing the shuttering to that required for the edges. The bottoms of the panels were splayed to form cutting edges during sinking and the splays later served as skewback supports for the cell bottoms. Splays were formed along the vertical edges of the panels to permit easy access of tools for welding the horizontal reinforcement.

Access through the cells for purposes of examination and cleaning out was provided by crawl-ways formed through the wall panels. During manufacture the crawl-ways were blanked off by plain concrete diaphragms, so that individual cells could, if necessary, be filled with water during sinking operations. The diaphragms were cut away after completion of sealing the cell bottoms.

Monolith Erection

The monolith walls were erected on plain concrete bearing-pads laid directly on the ground. Small square pads were used at the intersections of the pre-cast walls and continuous strip pads under the in-situ-poured external walls. Folding wedges and packing were employed to adjust initial levels of the walls and to counteract the small settlements of the pads which occurred during erection of the monoliths. The first monoliths to be erected were Nos 1, 2, 3, and 4 (see *Figs 4*), which form the power-station foundation. These were built on top of the general site filling. To reduce excavation costs and time of sinking, the remainder were built on the floor of shallow open excavations. The procedure adopted was to excavate the general site filling and about 3 feet of depth of the surface crust, then to build the monoliths on pads laid on a 6-inch-thickness of granular filling placed in the bottom of the excavation. Drainage of surface water was effected by rubble drains laid below the filling and connected to a pumping sump in one corner of the excavation. The open excavation and concrete bearing pads for monolith No. 6 are shown in *Fig. 8*.

Erection of the pre-cast walls was commenced at one of the central

cells, and panels were added, one at a time, until all internal walls of a monolith were erected. The panels forming the first cell to be erected were assembled around a timber templet frame which served temporarily to hold the panels vertical. The completed assembly of pre-cast walls for monolith No. 4 is shown in *Fig. 6* (*a*). The methods adopted for securing the walls during erection can be seen in *Figs 6* (*b*) and *7*.

The horizontal reinforcement projecting from the pre-cast walls was interlocked on erection and butt-welded. The welds were of the moulded and backed type using non-recoverable mild-steel saddle-pieces. The gap left between the bar ends to be filled by weld metal was $\frac{1}{2}$ inch wide and the bars used were $\frac{1}{2}$, $\frac{5}{8}$, and $\frac{3}{4}$ inch in diameter. The junctions between the pre-cast wall panels were concreted after completion of the welding. The 12-inch-thick external walls were then built in situ in lifts, a cutting-edge to match the splays on the pre-cast walls being formed at the bottom of the inside faces.

The whole of the outside of the peripheral walls was heavily coated with bitumen to improve watertightness and to afford protection against attack from sulphates, the presence of which in the soils had been shown by chemical analysis.

Sinking

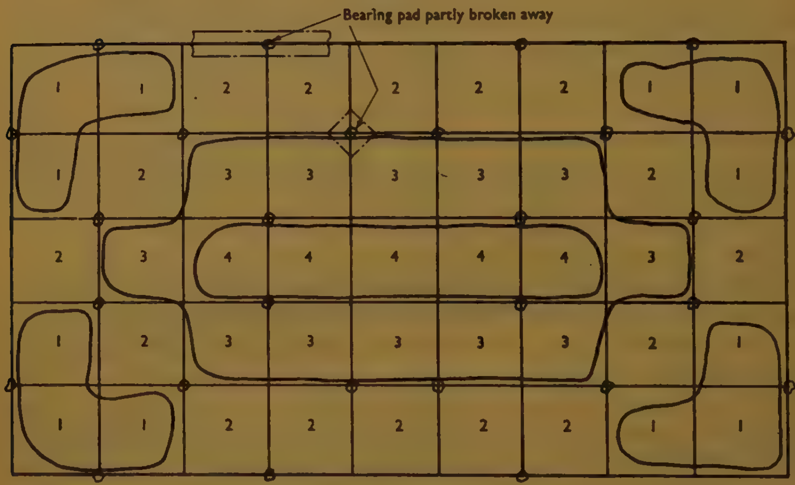
Sinking operations were commenced about a week after completion of monolith construction, when the concrete of the in-situ-poured walls and junctions between pre-cast wall panels was sufficiently matured to withstand the sinking stresses.

The concrete bearing-pads were broken away in stages to a predetermined pattern which provided uniform support of the monolith while the bearing area was gradually reduced. Downward movements commenced before the pads were completely removed. *Fig. 9* shows diagrammatically the final stage of cutting away the pads and the excavation stages for monolith No. 15. To avoid undercutting the walls, the excavation in each cell was worked to a dish shape, the bottom of which was not more than 1 foot below the cutting-edges except for a deeper grab hole at the centre. Approximately 2-foot stages of excavation were aimed at, commencing at the corner cells and working sideways and inwards. As sinking progressed, a further lead was developed at the corners and midway along the sides so that the excavation level in the central cells was about 2 feet above that of the peripheral cells at the end of a stage and up to 4 feet at its commencement. The excavation was controlled to keep a balance about the longitudinal and transverse axes so as to reduce the tendency to tilt. Throughout the sinking operations, a continuous check on line and level was kept and to correct any tilting which developed the excavation procedure was revised, as necessary, by deeper digging on the higher side to bring the monolith to an even keel. In all cases the weight of the monoliths was sufficient to cause sinking without the use of kentledge.

Some disturbance during excavation of soft soils is unavoidable, and to minimize this the last 3 feet of excavation was done by hand.

When nearing the final level, the downward motion was arrested by plugging the peripheral cell bottoms. The excavation in each cell was cleaned out and formed to an inverted dome shape. The plain concrete plug was then poured immediately, the top surface being formed to a more shallow inverted dome shape. As early strength was essential, the concrete for the plugs was made with rapid-hardening cement. Plugging of the bottom was continued, working from the peripheral cells towards the more

Fig. 9



MONOLITH EXCAVATION STAGES

I. Plain concrete bearing pads cut away to pre-arranged pattern
II. Excavation carried out in stages. Each stage commencing at 1 and working towards 4

central cells, the rate being controlled so that only a few central cells remained to be plugged when downward movement had ceased and the final level was reached.

Sealing of the cell bottoms was carried out after plugging. Protection of the reinforced-concrete seals against sulphate attack was provided in this case also by heavily coating the top surface of the plugs with bitumen.

The top surface of the seals was formed to a slight slope and holes cut through the internal walls, level with the top of the seals, afforded drainage paths to a conveniently placed sump formed in the bottom of one cell on each monolith. Ejector pumps were provided to clear periodically any water collecting in the sumps.

Structural Decks

The structural decks, consisting of pre-stressed concrete planks and in-situ concrete, were laid after sealing of the cell bottoms had been completed.

Cellular Foundation No. 7

This foundation was built in situ, inside a sheet-pile cofferdam, after completion of foundation No. 6. The piles were driven to the stony layer at about 30 feet depth. The overall width of the foundation was 12 feet. Proceeding from one end, the excavation was made and the plain concrete ground-mat and reinforced-concrete bottom-slab were poured in stages so that the bottom-slab provided a strut between the sheet-piling before excavation for the next stage was completed. The walls and structural deck were also completed within the cofferdam.

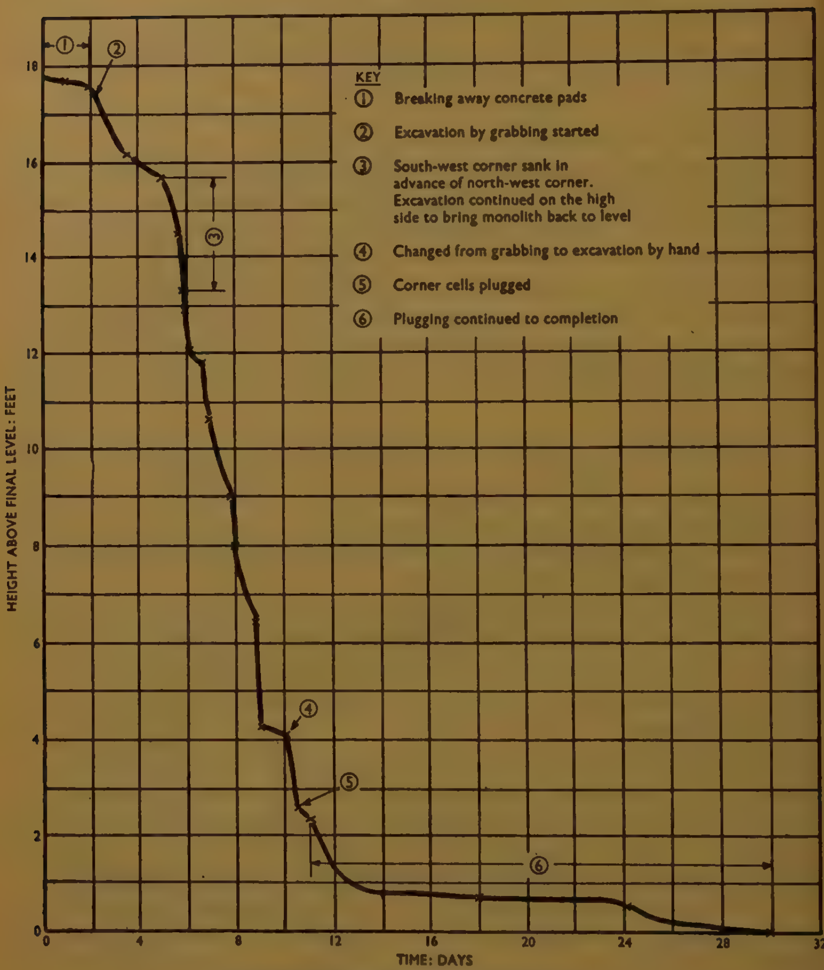
Notes on Sinking

Control of the line, level, and rate of sinking were found to be easier in the large monoliths than in the small ones. On an average the final positions were within 2 inches of alignment and 1 inch of level. Adjustments were made in the structural decks so that superstructures were set out correctly for alignment and level.

The excavation procedure shown in *Fig. 9* was adhered to for the majority of the monoliths, with minor adjustments to suit the different shapes and sizes. In small monoliths such as Nos 17 and 19, which have only six cells each, the four corner cells were excavated in advance of the two central ones.

The rate of sinking was to a large extent governed by the number of cells to be excavated and the construction plant available. In the power station unit, four large derricks capable of reaching the central area of the foundation were used; these were situated at the ends of the junctions between monoliths. Mobile plant was used in the refinery units since the monoliths were smaller and the plant was able to operate on more than one foundation simultaneously. *Fig. 10* shows the sinking record for monolith No. 15. This is typical of the refinery monoliths and indicates the rapid sinking-rate obtained during the period of grabbing, and the gradual slowing down of movement during the plugging of the cells. The fastest rate was 6 feet in one day, which was attained during the sinking of monolith No. 18. In the power-station monoliths, the average rate until commencement of plugging was about 1 foot per day. In some cases, where the sinking rate was high, excavation was carried well below the cutting-edges of the corner cells so that the plugs could be placed and the monolith allowed to sink on to them. This enabled the concrete of the plugs partially to mature before being subjected to stress. Even so, there was a number of cases where the first poured plugs fractured and had to be replaced after the remainder had been laid. This procedure was

Fig. 10



SINKING RECORD FOR MONOLITH No. 15

normally accompanied by a certain amount of soil squeezing through between the cutting edges of the outside walls and the plugs.

No actual sequence of sinking operations as between adjacent monoliths was laid down. Generally, the larger ones were sunk first and a close watch on the line and level of these was maintained during the sinking of others nearby. Sinking of monoliths was completed prior to commencement of work on adjacent surface-rafts.

In the power station, the monoliths were sunk in the order 4–3–2–1, with the work arranged to give an interval of about 2 weeks between

a stage on any monolith and the same stage on the next one. *Figs 1 (a)* and *1 (b)* show two stages during the sinking of the monoliths. The joining of these monoliths to form one foundation was started immediately after completion of bottoming operations. Sheet-piles, with walings packed off the monolith walls, were driven to close the outer ends of the junctions. Excavation proceeded from the sheet-piling towards the centre for the junctions between monoliths Nos 4–3, 4–2, and 2–1, and from the centre towards the sheet-piling for the junction between monoliths Nos 3–1, which was the last section to be done. Each junction was carried out in stages, the object being to complete the in-situ construction of the outer cell quickly and to continue excavation only when this concrete had matured. Subsequently, the work was staged in 10-foot bays so that the partially completed cellular work and the soil remaining to be excavated provided struts between the monoliths being joined, so as to lessen any tendency to unbalanced soil-pressure causing horizontal movement, and to minimize risk of upheaval of the bottom of the excavations.

In view of the experience gained during the sinking of the trial shaft previously mentioned, it was anticipated that there might be some tendency to inflow of water or soil under the cutting-edges of the external walls, particularly when sinking through the permeable shelly layer and when nearing the final levels. Serious cases of such inflow would have been controlled by immediate filling of the affected cells with water to balance the pressures, and excavation would then have continued in the wet until further sinking of the walls formed an effective seal. Inflow during plugging could have been controlled in similar manner, the plugs being placed under water with pressure-relieving standpipes passing through them. Although some inflow did occur, the amount was comparatively small and control was maintained without resort to flooding of cells.

That so little trouble was experienced from inflow of water is explained by the low permeability of the soils, and by the control exercised on the excavation level in relation to the cutting edges. Generally, the soils in the refinery sites were firmer and appeared to have a lower water-content than those in the power-station site.

When the sinking of No. 3 monolith was nearly completed there was an inflow of water to the peripheral cells on the north side. These cells were pumped out and plugged immediately. Water inflow was again experienced in the north-west corner cell of monolith No. 2 when nearing the completion of sinking. A well-point tube put down outside the monolith was sufficient to clear the water while the north-west corner and adjacent peripheral cells were plugged.

The following cases of soil inflow are worthy of note :—

(1) A small inflow of soil into the cells occurred at the south-west corner of monolith No. 2 when within 1 inch of the final level. This was stopped by immediately plugging the affected cells.

(2) Monolith No. 6 was about 6 inches above the final level and cleaning-out of the corner cells on the south side in preparation for plugging was proceeding when the monolith began to sink rapidly. Plugs were placed immediately in the corner cells on the south-side, which was sinking more rapidly than the north. Although these cracked they were beneficial in helping to arrest movement. The south side sank 11 inches and the north side 4 inches in 11 hours. The movement continued more slowly and the monolith became stable again after 4 days, by which time the south side had sunk 15 inches and the north 9 inches. The cells on the south side were then plugged, the fractured corner cells were replaced and the monolith was allowed to sink under its own weight until the top was practically level, after which plugging of the remaining cells was completed. The monolith was then $7\frac{3}{4}$ inches below the designed level and the walls were subsequently built up to correct this. The movement of this monolith probably started as a result of soil inflow on the south side, where excavation in the corner cells was dished downwards from the cutting edges of the outer walls in preparation for plugging. The inflow was quickly sealed off as the cutting-edges sank into the soil.

(3) Local patches of softer soil occasionally caused temporary loss of alignment. A typical case was monolith No. 11, which tilted 2 feet 9 inches in 27 hours owing to a soft patch of soil under the north-east part of the monolith. The monolith was about 5 feet 6 inches above final level when the tilting started and by faster digging on the higher side it was brought to within 1 inch of level during the next 3 feet 6 inches of sinking.

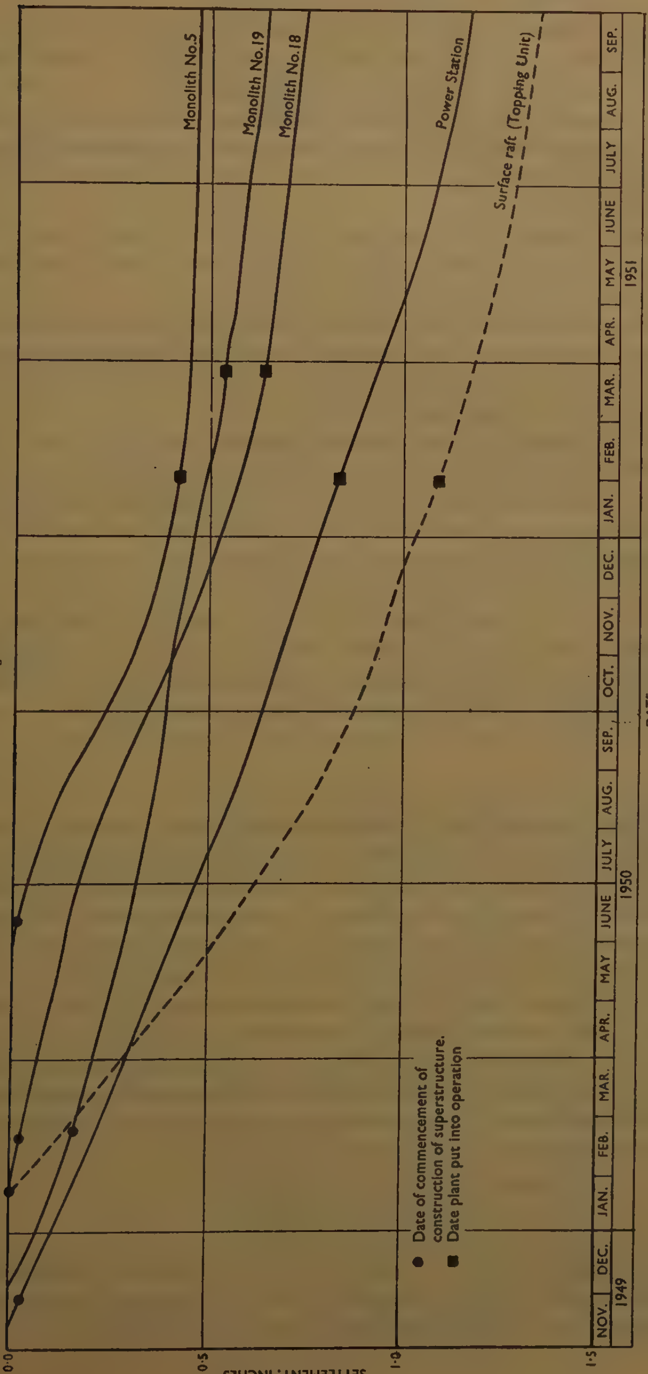
Some leakage of water occurred through construction joints between the outside walls and bottom seals, but pressure grouting in the joints ensured that all foundations were made reasonably watertight.

Settlements

Settlement of the whole site resulting from natural time consolidation, which must be taking place in the Grangemouth soils, is not of practical importance when considering the foundations, since in this respect both site and foundations will settle equally. The question of relative settlement between individual foundations and adjacent ground does, however, require more careful examination.

In an ideal case, a buoyancy raft should not settle relative to the surrounding ground, because the equilibrium conditions of the founding layer are theoretically the same on completion as before commencement of the construction. As previously mentioned, however, some disturbance and possibly upheaval of the bottom of the excavation are unavoidable during construction owing to the low shear strength of the soil. Hand excavation during the last 3 feet of sinking minimized the soil disturbance, and the tendency to heaving was hindered by the cellular form of the monoliths.

Fig. 11



FOUNDATION SETTLEMENT CURVES

The equilibrium conditions of the founding plane cannot be restored until the working load has been added to that of the foundation, that is, until the construction of the superstructure or plant supported is completed. Some settlements are to be expected during this construction period and will continue afterwards until the soil has recovered its capacity for passive resistance, which must have been to some extent destroyed by the disturbance during excavation.

Previous mention has been made of a series of reinforced-concrete surface-rafts that was used for founding the lighter load-concentrations. These were designed for a uniform soil bearing-pressure and were founded as near the surface as plant requirements permitted, so as to take maximum advantage of load-spread through the general site filling and the tougher surface crust. These foundations will, of course, be subject to some settlement owing to compression of the ground.

It was estimated that probable settlements during the lifetime of the plant would be about 1 inch for the buoyancy rafts and about 3 inches for the surface rafts. Allowance for this was made in the initial foundation levels by siting the surface rafts 1 inch higher than the buoyancy rafts, so that eventually the surface rafts may be 1 inch or so below the buoyancy rafts.

Fig. 11 shows settlement curves to date for some of the first completed foundations. Curve E is for a surface raft and the others are for buoyancy rafts. Settlements of the remaining foundations, not shown in *Fig. 11*, appear to be following the same general trend.

Conclusions

Although the principle of using net soil bearing-pressures is firmly established, its use in the limiting case, where net pressure is reduced to zero and the foundation becomes earth buoyant, is rare. The Authors believe that it has not been previously developed on the scale employed at Grangemouth. A total superstructure and plant load of about 37,000 tons has been carried on a series of fifteen separate foundations, which range in size from about 170 feet by 170 feet down to 33 feet by 22 feet 6 inches. The maximum depth of excavation was 24 feet 6 inches, for monoliths Nos 12 and 13, the top of these being 4 feet 6 inches below ground level. This is considerably in excess of the depth to which open excavation could have been made. Some settlements were envisaged from the outset and a close watch will continue to be kept on these.

Acknowledgements

For permission to produce this Paper the Authors are indebted to the Anglo-Iranian Oil Co. Ltd, Scottish Oils Ltd, and Messrs Rendel, Palmer and Tritton, the Consulting Engineers for the works described.

The principal Contractors were George Wimpey and Co. Ltd, and the soil investigations were carried out by Soil Mechanics Ltd.

The Paper is accompanied by seven photographs and seven sheets of drawings, from which the half-tone page plates and the Figures in the text have been prepared.

Discussion

The Authors introduced the Paper with the aid of a series of lantern slides.

Mr A. C. Hartley observed that when any large enterprise had to be planned, the site should be chosen from very many aspects. The commercial points of view, which would be operative throughout the whole existence of the enterprise, were perhaps fairly predominant. In the case of an oil refinery, the site had to be suitable for bringing in the crude oil and have assured availability of power, water, and the labour required to operate the plant. It had to be well placed from the point of view of road, rail, and sea transport, to export the products of the refinery. As each enterprise was built, it might be assumed that the best available sites were chosen, so that in later years it became more difficult to find a suitable site and more and more impossible to find all the conditions that were wanted. Moreover, the unseen conditions—those below the ground—were liable to be forgotten at first.

In the case in question it had been known that the site was a very difficult one. As the Authors had explained, the refinery had been there for nearly 30 years, but the loads due to each new plant had tended to increase. Settlement could no longer be tolerated between parts of modern units. There were very large high-pressure high-temperature pipes which connected the units and sections of units, and there was a definite limit to the settlement which was tolerable. The Paper indicated, therefore, one of the greatest services that civil engineers could render to industry—finding ways of doing the almost impossible and providing a reasonably economic foundation in such difficult conditions.

In the case in question, a very great service has been done by the soil mechanics experts and by the early investigation of the site. The Anglo-Iranian Company had made use of their geo-physical exploration facilities and had been able to get a very good underground contour plan, as shown in *Fig. 2* of the Paper. It gave an opportunity of assessing, in the near future, by watching settlements and so on, at what contour it paid to change, other things being equal, from piles to buoyant rafts. He did not think that there was any doubt that it had been right to use, in the associated Company, piles when there was boulder clay at a depth of about 80 feet,

and he felt certain that there was no choice but the buoyant raft at 200 feet; but it would be very interesting to try to get an assessment in due course of whether it was at 80 feet, 90 feet, or 120 feet that consideration should be given to the question of whether to use piles or buoyant rafts.

He did not think that there need be any doubt about the full effectiveness of the buoyant raft, because the precautions taken at Grangemouth had ensured that the soil below a depth of 30 feet was not disturbed; the soil was therefore unlikely to give way, and the buoyant raft certainly would not, so that the founding could be regarded as safe and certain.

The founding gave much the same effect as that obtained in certain pump-houses in Abadan, right alongside the river. The pumps in that case had to be put in the bottom of the foundation, because a drowned suction to the pumps was wanted; but in effect that was a large watertight chamber which displaced about as much soil in weight as the weight of the pumps, turbines, and condensers that had been installed at the bottom. In that case the foundation was very near the river, and for a time at any rate it moved very slightly according to the rise and fall in the Shatt-el-Arab which flowed beside it.

Dr H. Q. Golder observed that one of the greatest difficulties in site investigation work was to get the client's engineer to refrain from protesting at the very beginning that the work would be too expensive. On the site at Grangemouth, however, there had been no question of that at all, because, as the Authors had said, there were five chimneys on the site and two of them had settled 12 inches, and had developed a considerable tilt in so doing.

The Authors—or perhaps one should say the engineers responsible, because there were, no doubt, others involved—had adopted a courageous attitude in regard to what might be called a new-fangled idea. After the site investigation had been finished, he had been most interested to see what they were going to do, because it was a very difficult problem. The Authors' suggestion that it was the biggest buoyant foundation so far made in Great Britain, was probably correct, but it occurred to Dr Golder that when a dry-dock was being built, something of the same effect was obtained. The dry-dock, when empty, was probably floating, or almost floating. At any rate the Authors had adopted the expedient described, and it worked.

An expedient, however, was not a success simply because it worked. Engineering was more difficult than that. In engineering, for an expedient to be a success, it had to do something either more cheaply than any other means, or more quickly, if time was important; or it was used because it was impossible to do the thing at all by any other means. He would like the Authors to classify the case which they had described into one of those three divisions.

He would also like them to give, if they could, some idea of the cost of the expedient. He was not trying to find out the contractor's price for

the job, but the Authors might be able to give an idea of the price per ton carried, or something of that kind, because many engineers kept such a figure in their head for other methods; piles, for instance, might be £2 or £3 per ton carried, though in the case in question the figure might be considerably greater.

On p. 306 of the Paper the Authors had referred to some tests which had been carried out with steel box-piles on an adjoining site, which showed that individual piles were capable of carrying a working load of 60 tons. How had that working load been determined? Had the piles been loaded up to 60 tons, or something more, and 60 tons estimated as a safe load, or had any of them been loaded up to a condition approaching failure?

On the same page, the Authors stated that steel or concrete cylinders had not been considered. Had Benoto concrete cylinders, 5 feet in diameter, been available at the time? Were the Authors familiar with them?

On p. 307 reference was made to a trial shaft, 8 feet in diameter. Would the Authors expand on what they had learned from that and say why, when water flowed in, they had pumped it out; he would have thought it was just as easy to grab under water, and probably a good deal safer. That question came up again on p. 317, where reference was made to water flowing in and being pumped out. The cells had been pumped out and plugged immediately, but would it not have been safer to plug them with the water in the cell? For placing those plugs was concrete poured in the bottom of the hole, and, if so, how much time elapsed after placing before any load was applied to the plugs?

On p. 320, under the heading "Settlements," the Authors used the phrase "until the soil has recovered its capacity for passive resistance. . . ." He thought that that was slightly misleading, and that the settlements which occurred were partly elastic movement—as the load which had been taken off the soil was replaced—and possibly some consolidation in the later stages, because it was obviously impossible to balance exactly over the area of the building the loads applied with the soil taken out. There would be some consolidation caused by disturbance of the soil and by the fact that the pressure/voids ratio curve for the disturbed material was probably slightly lower than that for the natural material.

Dr A. W. Skempton observed that the Authors had described an extremely interesting application of buoyant-foundation design. There was little doubt that that principle was being increasingly used, not only for difficult sites such as Grangemouth, but also in those cases where economy could be gained by combining a raft with one or more basements, thereby effecting a reduction in net bearing pressure and settlement. The idea of using buoyant or partially buoyant foundations was not of recent origin. It was said [2] to have been employed by Rennie in building

[2] A. Casagrande and R. E. Fadum, "Application of Soil Mechanics in Designing Building Foundations." Trans Amer. Soc. Civ. Engrs, vol. 109 (1944), p. 383. *Discussion* by K. Terzaghi, p. 427.

the Albion Mills, in London, in 1784. But the systematic application of the method seemed to date from the later 1920's. A well-known example was the Albany Telephone Building [3] in New York, built in 1929, where the columns and beams of a two-storey basement, and the foundation raft, were combined to form a Vierendeel truss, thus providing a rigid and comparatively light sub-structure, with a very considerable reduction in net bearing pressure as compared with that imposed by a surface raft.

The largest application of completely buoyant foundations, where the weight of excavated earth equalled the structural load, was the New England Life Insurance Building in Boston,[4] built in 1939. That building covered an area of 200 feet by 340 feet and weighed about 130,000 tons. The excavation had been carried 35 feet below street level and two basements had been constructed. In spite of the enormous load, and the fact that the ground consisted of only moderately firm clay to a depth of 80 feet below foundation level, the total settlement to date was not more than about 2½ inches, and had practically ceased.

That settlement was approximately equal to the heave that had occurred during excavation. At Grangemouth the monoliths had settled between ½ inch and 1¼ inch, and the Authors attributed that to "some disturbance and possibly upheaval of the bottom of the excavation." Moreover, they stated (p. 318) that "In an ideal case, a buoyancy raft should not settle relative to the surrounding ground. . . ." That was not correct, however, since the removal of load by excavation was bound to cause an expansion or upheaval. Conversely, when the load was replaced by the structure the soil would return more or less to its original position. But that return to the original position was, in fact, a settlement—a settlement that could not be avoided even in "an ideal case."

It was therefore very desirable to obtain as much information as possible on the settlements taking place under the replacement of the excavation load. A few such field records existed and the data given by the Authors formed an important further contribution. In order that the information should be as complete as possible, would the Authors add to Table I the plan dimensions of the monoliths?

It would also be helpful if a more representative strength/depth relation could be given than that in *Figs 3*. The upheaval in the excavation was related to the ratio of overburden pressure to shear strength. At foundation level the shear strength was, according to Dr Skempton's investigations,[5] about 300 lb. per square foot and did not vary so markedly

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[3] G. W. Glick, "Foundations of the New Telephone Building." Proc. 1st Int. Conf. Soil Mech. & Foundn Engng., 1936, vol I, p. 278.

G. P. Tschebotarioff, "Soil Mechanics, Foundations, and Earth Structures." McGraw-Hill, New York, 1951, p. 368.

[4] A. Casagrande and R. E. Fadum, "Application of Soil Mechanics in Designing Building Foundations." Trans Amer. Soc. Civ. Engrs, vol. 109 (1944), p. 383.

[5] See footnote 1, p. 305.

as shown in *Figs 3.* Also the strength below 30 or 40 feet undoubtedly increased with depth and, in that respect, *Figs 3* was most misleading. Taking a strength of 300 lb. per square foot, the clay would be approaching failure when the monoliths were nearing their full depth. The upheaval, which was, of course, taking place in the ground within a depth of many feet below excavated level, would therefore be rather large.

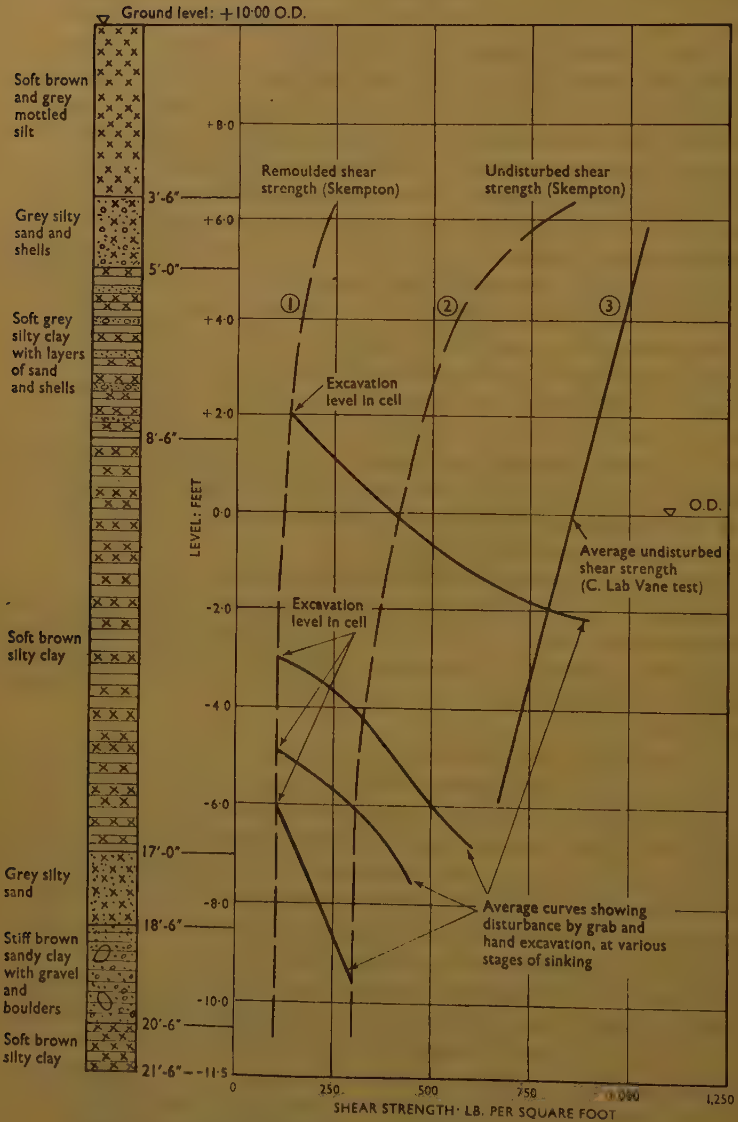
Consequently it might well be that, had the monoliths been sunk to a somewhat shallower level, the settlements would not have been greater than those observed, since although there would then have been a net positive foundation pressure, the upheaval would have been smaller. In that way some saving in cost might have been achieved. Finally, it would be most valuable if the Authors could give the settlement curves for all or most of the monoliths and two or three of the surface rafts (with their actual loadings), with observations extending up to, say, September 1952. That would be particularly useful, since it might be expected that the settlements would be almost completed by that time.

Dr L. J. Murdock gave an analysis of the conditions obtaining during the sinking of the monoliths. A number of vane tests had been made on the site during the grabbing and sinking operations, and the shear strengths so obtained had been compared with the estimated shear stresses set up in the soil during the sinking of the monoliths. The results of the vane tests were shown in *Fig. 12.*

The soil strata were outlined at the left of the Figure. Curve 1 showed the remoulded shear strength given by Dr Skempton and later confirmed by the tests made during the site works. Curve 2 showed the undisturbed shear strength given by Dr Skempton, and Curve 3 the undisturbed shear strength as obtained by the vane test near the monoliths at the time of sinking.

At four stages during the excavation, vane tests had been made from the excavated level to a depth of several feet. By plotting strengths at the respective depths, four ancillary curves had been obtained, commencing in every case from the curve for remoulded shear strength, and becoming coincidental with the curve for the undisturbed vane shear strength about 4 feet below the excavation level. It was, therefore, evident that the disturbance during the sinking had been complete at excavation level and had become progressively less, disappearing at about 4 feet below the surface exposed. It was thought that the disturbance had been caused not only by the mechanical effort of removing the soil, but also by the movement of the soil below excavation level arising from the downward movement of the monolith. Dr Murdock suggested, therefore, that even when the monolith had reached its final position, there had been a zone of partly remoulded soil immediately beneath it, which might have taken some little time to reach a new equilibrium condition, and thus accounted for some portion of the small settlement shown in *Fig. 11.* The extent of any heave mentioned by Dr Skempton, or swelling as the overburden load

Fig. 12

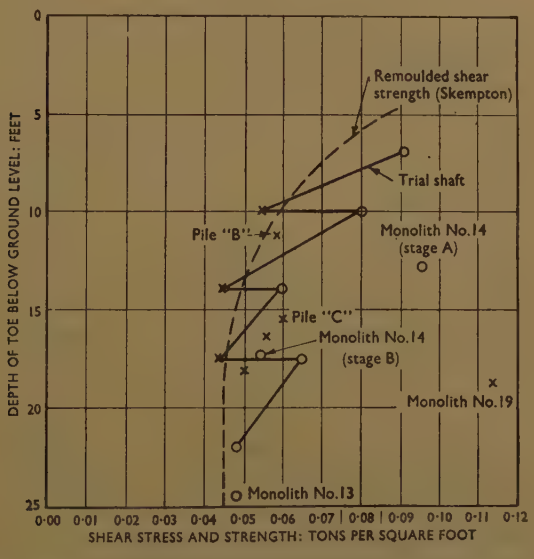


Shear Strength of Soil at Power-Station Monoliths

was removed during sinking was, Dr Murdock believed, limited considerably by several factors, including the method of excavation in stages (as indicated in *Fig. 9*), the presence of the stiffer clay layer at a depth in the region of 20 feet, and the rate of sinking as indicated in *Fig. 10*, coupled with the general impermeability of the deeper strata.

From the evidence he had given, Dr Murdock thought it would be apparent that the soil immediately adjacent to the monoliths might be taken as completely remoulded, and on that basis an analysis had been made to show the relationship between the shear stresses set up in the soil immediately adjacent to the surface of the monolith, and the remoulded vane shear strength. *Fig. 13* showed that relationship for the trial shaft,

Fig. 13



SHEAR CONDITIONS DURING SINKING OF TRIAL SHAFT, MONOLITHS, AND PILES

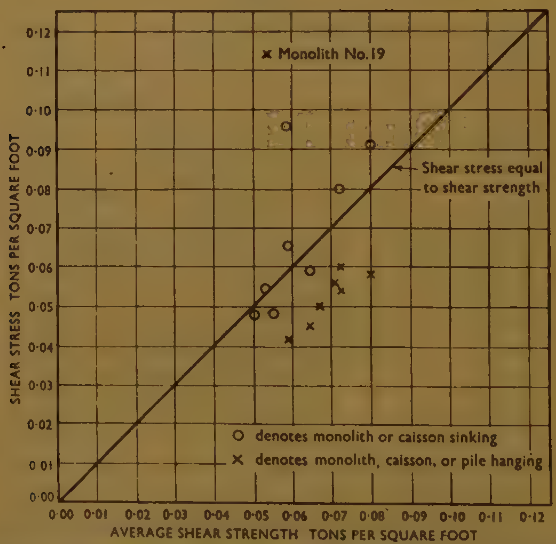
described by the Authors on p. 307, and for three of the monoliths. The remoulded shear strength was shown by the broken line, whilst the estimated shear stresses were indicated by the circles and crosses. In each case where a circle was shown, the downward movement of the trial shaft or monolith had been progressing satisfactorily, whereas, at the points indicated by the crosses, sinking had slowed down or ceased and kentledge had had to be added. Two points were also included for test piles on the adjoining site, which had sunk under their own weight together with that of the pile hammer resting on the pile head.

The levels at which kentledge had been added and movement had started again were clearly shown by the horizontal lines. The cross indicated for monolith No. 19 was clearly an exception to the general rule. That was

the only monolith which had hung-up, probably because of a delay during the sinking of that particular unit. The results had been replotted in *Fig. 14* to show more clearly the measure of agreement between the shear strengths and estimated shear stresses. The points for cases where the monolith or test piles had been hanging, except for monolith No. 19, were all below the line of equal shear strength and shear stress, whilst most of those for the sinking conditions were either very near the line, or above.

The results indicated that the sinking force required to maintain the movement of monoliths, caissons, or similar structures in such soil could be estimated fairly accurately from a knowledge of the remoulded shear

Fig. 14



RELATIONSHIP BETWEEN SHEAR STRESS IN THE SOIL AND ITS STRENGTH AS DETERMINED BY THE VANE TEST

strength of the adjacent soil. In calculating the required sinking force, due allowance should be made for buoyancy of the buried portions of the monolith wall, and for bearing resistance if the cutting edges were not to be fully undercut during the sinking. That method of analysis might be of considerable value during both design and construction of those types of foundation structure, for assessing sinking conditions and for estimating the weight of any kentledge which might be required.

Mr J. McHardy Young observed that the problem which had faced the Authors at Grangemouth was very similar to one which had arisen in connexion with the design of the Ijoka "B" Power Station, at Lagos, Nigeria. It might be of interest to quote one or two figures. In the latter case the shear strength was of the order of 200 lb. per square foot, which appeared to be very much the same as at Grangemouth. The

load from the superstructure was a little more than 0·5 ton per square foot, and the total load, including the foundations, was about 0·9 ton per square foot. That gave a depth of footing, to be fully buoyant, at 17 feet, assuming 120 lb. per cubic foot. The fully buoyant foundation, in his opinion, was the only possible solution for cases such as Grangemouth and the particular power station to which he had referred.

On the question of design, on p. 311, the Authors had stated that "The walls, bottoms, and structural decks forming the foundations were designed to resist (in combination) the main raft stresses, in addition to the stresses set up by the local loading." When some of the photographs of the actual work were examined, it was found that there was no thickening of concrete where the walls intersected, and therefore it would appear that the loads from the columns must be carried to the walls and through the walls to the slab forming the bottom of the foundation. Mr McHardy Young would like to ask the Authors what provision was made for carrying those loads through the walls. It seemed to him, from a practical point of view, that the walls were extremely thin, bearing in mind the fact there was a maximum depth of 20 feet and that the cross-walls were at 10-foot centres. He thought that the behaviour, particularly in bending, would be very problematical.

He would also like to ask the Authors about the provision for shear. The reinforcement of the walls was shown in *Fig. 7*, and the horizontal reinforcement seemed to be reasonably heavy and to be spaced closer together near the top and bottom. On the other hand, the vertical or shear reinforcement seemed to be lighter than the horizontal, and seemed extremely light. He would like to know whether in fact any cracking had been observed in the walls.

One point which had to be borne in mind when considering the walls and the reinforcement was that, in a power-station basement, provision must be made for a large number of pipes. He agreed that at Grangemouth that had been done to a certain extent by providing alleyways, but there might be other pipes which could not be grouped in an alley and which would tend further to reduce the strength of those walls in shear.

The Authors had stated that they preferred the monolith to the open excavation in order to reduce heave. Did they experience any heave during sinking? They had also stated that kentledge was unnecessary and that the walls had sunk under their own weight, but had there been any tendency to run during sinking?

Finally, there was the question of differential settlement. It was well known, from a practical point of view, that even in very bad ground it was possible to get hard spots. Had any provision been made for differential settlement in designing the foundation and, if so, how much?

Mr John Cuerel observed that, although he was not a soil mechanics expert, in his view the vane tests gave a truer indication of the soil strength when the soil was actually undisturbed; the triaxial-test results which

were plotted in *Figs 3* represented the disturbed strength of the soil, notwithstanding the fact that very often the samples taken were described as undisturbed samples. Engineers were, of course, interested also in the disturbed strength of the soil, because that was often what they had to contend with during construction, and there was no doubt that the particular soil was very sensitive to disturbance.

Reference had been made in the Paper to a power-station job in India. That foundation was considerably larger than any at Grangemouth, its dimensions being about 330 feet by 230 feet, and it had been possible to have an open excavation. The maximum gross pressure of more than 1·5 ton per square foot had been reduced to a net pressure of a little less than 0·5 ton per square foot by taking away the soil and substituting a cellular base. The rise of the formation had been measured and amounted to 3½ inches at the centre, with an average of 3 inches or a little less, and allowance had been made for that. When the 2-foot bottom slab had been laid, ½ inch or more of swelling had been taken out, merely by the weight of that bottom slab of the cellular work. When the foundation had been completed, the superstructure erected, and the plant installed, the settlement amounted to a bare 3 inches. The power station had been in commission for about 2 years, and the settlement subsequent to commissioning had been extremely small. He did not think that it would amount to very much in the future.

At Grangemouth, owing to the speed with which the work had to be done, it had not been possible to measure the rise of the formation, but, as anticipated, the same thing had happened; the ground had swollen, and that swelling had to be taken out before full stability was reached. The power-station foundation with its alleyways took longer to construct and so there had been much more time for swelling to take place. In the case of No. 6 monolith it had been thought reasonable to take a chance and bottom-up the whole thing almost all at once; but there had been a certain amount of piping, the monolith had dropped down, and that had led to more disturbance. The outcome had been that the power-station and No. 6 monolith settlements were greater than the others, because of the extra dilation of the underlying soil.

With regard to the principles of design, it was assumed in the case of an ordinary bridge monolith that the hogging or sagging effect was represented by a support in the middle or supports at the ends. At Grangemouth that would have meant a good deal of additional reinforcing steel, and the assumption was made that the support would be peripheral or over the centre quarter of area of the monolith. He did not know whether that assumption had still been too conservative, but no cracks due to hogging or sagging stresses had been noticed, which would perhaps answer the question of whether the shear reinforcement had been sufficient.

He felt that perhaps a mistake had been made with the power-station monolith from a purely technical point of view, in that it had been divided

into four; more boldness, making it 170 feet square and putting it down in one, might have paid better. He suggested that the central walls on the lines of the alleyways could have been made a little weaker. When the monolith was sunk certain cracks would have occurred, but it would have been less difficult to deal with those than to close the gaps which had been deliberately provided.

The conventional way of starting-off a monolith was to construct it on sleepers and pull out half the sleepers and then half of what were left and so on, but at Grangemouth concrete pads had been adopted. He did not intend to use sleepers again, because the pads had been most satisfactory.

Reference had been made to costs. He had understood that, for the later units, when the job was running smoothly, the monolith cost had been brought down to the equivalent cost of a foundation on 70-foot steel piles.

Dr Golder had asked about the purpose of the trial cylinder. It had several purposes. The contractor had wanted to know whether it was necessary to order kentledge, and if so, how much. The consulting engineers had felt that none would be required at all. The trial cylinder had actually needed a small amount of kentledge, but the weight per unit area in contact with the soil had been less than was available with the bigger monoliths and it had been proved that no kentledge would be required for those. Another use had been to get some idea as to whether piping was likely to occur, sinking in the dry. None had occurred in the cylinder, which had gone down more than 20 feet, and so it had been made evident that no serious piping need be feared.

Personally, Mr Cuerel did not like Dr Skempton's suggestion that the depth of a foundation should be limited for fear of piping. There was a simple solution, which was to let the water in and sink "in the wet" to any practical depth desired.

Mr D. H. Little recalled that an approach had been made to the Building Research Station in 1942 for assistance on the soil side of the design of a graving dock. That dock was a big one—comparable with the big ones which were built to-day—1,400 feet long overall, about 200 feet wide overall, and about 70 feet deep. The amount of earth removed would have been over 800,000 tons; the weight of concrete in the finished dock would have been 350,000 tons, and the weight of water when the dock was flooded about 350,000 tons. That was the maximum load that the structure would take. When the ship was in the dock and the water out, the ship would weigh 60,000–80,000 tons. If, therefore, one took away 800,000 tons, the most that would ever be put back would be 700,000 tons. He thought that most dock engineers appreciated that graving docks floated in the ground and the first reaction of Dr Skempton, who had been at the Building Research Station at that time, had been that there was no problem, except to hold the structure down. Mr Little suggested that whilst

the Paper might describe the biggest foundation which had been deliberately put down on those lines in Great Britain, all graving docks were in effect buoyant foundations and they acted as such.

He would like to ask two questions on the Paper as a whole. What had been the weight of earth taken for the design of the foundation? Did the cells have to be kept dry for the foundation to be a success, and, if that were the case, what would be the effect on the foundation if the pumping arrangements broke down? Was it definitely a fact that any water leaking in had had to be pumped out? Had the base been designed for the hydraulic uplift or for the uplift of something heavier than water, namely the 90 or 100 lb. per cubic foot which those silts weighed? Had the uplift been taken from any water-table level or from the level of the surface of the ground? His own experience of those silts and the testing of them covered Rosyth, along the Clyde, Plymouth, Sheerness, Belfast, Sierra Leone, and Shanghai. He thought that all of them had the same characteristics, including a shear strength of 150–200 lb. per square foot, and all, when piles were driven into them, seemed to develop an increased shear strength of about 5 cwt per square foot with regard to carrying capacity for friction piles. He thought that a figure of 5 cwt per square foot would give the gross load at which the pile began to fail. The piles in question, if 100 feet long, might have been expected to take a load of about 100 tons. On what had the figure of 60 tons been based?

Dr T. P. O'Sullivan asked whether the Authors had given any consideration to the possibility of using a piled raft, either a piled surface raft or a piled buoyancy raft. It would appear, he said, that the provision of some piles in the case of a buoyancy raft would be of assistance in two ways. First, those piles should help to reduce the heaving effect caused by the removal of the earth, and also they should minimize the consolidation effect. Thus, by the provision of relatively short piles, the settlement which took place in such circumstances would presumably be reduced. Alternatively, a buoyancy raft of smaller dimensions might be used if piles were incorporated in the base, to give the same results.

Mr E. M. Barnes asked why the cells had been made only 10 feet square. He thought that if they had been made, say, 15 feet square, the cost would have been reduced; he knew that, in grain silos, bins which were 14 feet square were cheaper than those only 10 feet square. He was really thinking of a caisson constructed in situ rather than one of pre-cast units. With cells larger than 10 feet square, the walls could have been thicker, if it had been desired to have a certain weight for sinking, and this would have resulted in cheaper rates for placing of the concrete; also there would have been fewer square yards of shuttering. Had the cells been made 10 feet square because of a desire to pre-cast the units in order to save shuttering? Would it not have been cheaper if the caissons had been cast in situ with cells of larger size?

Mr B. F. Saurin, in reply, observed that the cost per ton had not

been mentioned and it was not thought that that would be of general use, but an indication had been given of how the buoyant-raft design compared with the piled design in circumstances where the latter could be used. In considering comparative estimates for future work it should be remembered that the buoyant-raft method did require the building-up on site of organization, skill, and equipment and only a comparatively large work would permit advantage to be taken of that development. On the other hand, where site conditions made a piled design possible, work could go ahead immediately on well-known lines with equipment and organization readily available.

The 60-ton working load on B.P.3 box piles about 100 feet long which had been used on an adjacent site had been, if not determined, at least covered by a 120-ton test load. The engineers responsible for that work had specified that the piles should be driven to a certain set in order to make sure that they were in the boulder clay. The piles were consequently end bearing piles and not friction piles. The 120-ton test load had produced practically no permanent settlement.

Reference had been made to cylinders sunk with the Benoto hammer grab. At the time the foundation scheme had been determined that system of construction had not been known. The use of cylinders had been suggested but the distribution of loads in that case had made their use inappropriate.

With regard to the shear strength of the very soft clay Mr Saurin thought that it was of the order of 200–300 lb. per square foot, perhaps 230 lb. on the average. At the time the soil investigation had been made the vane test had not been developed and he was happy to say that it was at the Grangemouth Refinery site that Dr Skempton and Soil Mechanics Ltd had done some of their development work on the vane test apparatus. If the work had to be done again reliance would be placed on in-situ tests such as the vane test. In fact, in that particular work, the shear strength of the ground had been of secondary importance only, but if it had been a case where the shear strength of the ground would rule the choice between, for instance, compressed air and open excavation an interesting administrative point would have been raised. A figure for the shear strength of approximately 100 lb. per square foot had been obtained by the best methods of testing available at the time but that low figure had never been accepted without reserve and the later results showed that the general feeling that a much higher figure would be correct had not been wrong. If, however, there had been some difficulty over the foundations in such a case and an engineer had to justify his design it would have been very difficult for him to produce the evidence on which he had based it. It would appear that he had taken a great deal of trouble to get experimental results to show the shear strength of the ground and had then adopted a design based on a surmise that those experimental results were too low.

Mr C. W. Pike, in reply, thanked Mr Cuerel for dealing with a number of the points raised.

The aim of the design had been to produce a foundation having the smallest possible weight per cubic foot of space occupied. By adopting cells 10 feet square in the clear it had been possible to use pre-cast internal walls 6 inches thick and to keep the top and bottom slabs within reasonable proportions.

The pre-cast wall panels had proved very convenient in manufacture and in handling during monolith construction. Timber had been in short supply at the time and by casting the panels flat, in tiers, with building paper between, the amount of formwork had been reduced to that necessary for the edges. The reinforcement had been threaded through holes accurately drilled in the formwork and when the panels were positioned in the monoliths the specified gap of ½ inch between the projecting bar ends had been maintained within very close limits. The panels when built into a monolith were rigidly held along all four sides and therefore were not comparable with slender columns subjected to bending.

The weight of the soil taken from the site varied between 108 and 118 lb. per cubic foot and an average value of 115 lb. per cubic foot had been used in the design. The unit pressure beneath the foundation was that due to a column of soil of height equal to the distance from ground level to the bottom of the monolith.

Some thought had been given to the use of piled rafts for both surface and buoyant foundations. However, with both types it had been possible to arrange plan dimensions suited to the size of the supported structures. That could be taken as reasonable indication that additional support from piles was unnecessary. Further, the aim throughout had been to cause a minimum of disturbance to the soils beneath the foundations and that condition could not have been satisfied had piles been used.

The design assumed that the cells would be kept dry and automatic ejectors had been installed to deal with seepage water. The monoliths were not absolutely watertight but the seepage which had occurred had been of the order of ⅛ inch per month, which was considered a reasonable degree of watertightness. Failure of the ejectors would result in serious overloading of the foundations only after a very long period, and since arrangements had been made for regular periodical inspections it seemed unlikely that such a condition could arise.

Correspondence on this Paper is closed. No contributions may now be accepted.

WORKS CONSTRUCTION DIVISION MEETING

1 April, 1952

Mr David M. Watson, B.Sc., Vice-President I.C.E., Chairman of the Division, in the Chair

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Authors.

Works Construction Paper No. 21

"The Construction of the Caisson forming the Foundation to the Circulating-Water Pump-House for the Uskmouth Generating Station"

by

William Storey Wilson, M.C., B.Sc., and Frederick William Sully, MM.I.C.E.

SYNOPSIS

The Paper describes the design and sinking of a caisson, measuring 164 feet by 110 feet in plan and 80 feet deep, which forms the foundation to the pump-house for the circulating-water intake for the Uskmouth generating station, near Newport, Monmouthshire.

The steelwork forming the caisson shoe and working-chamber was of all-welded construction, and the working-chamber was divided into three compartments, of equal size, by the provision of two internal transverse bulkheads accommodating spine trusses into which were framed the roof girders over the working-chambers. The roof plating to the working-chamber was cambered to form vaults, and the outside plating of the caisson extended to a height of 20 feet 9 inches.

The caisson shoe, weighing 510 tons, was erected on ball-carriages, behind the river bank, and rolled out on two tracks on the line of the bulkheads to a position above high tide and immediately above the site on which it was to be sunk. From this position it was lowered by hydraulic jacks to a prepared and level hardcore bed on the foreshore.

The caisson was provided with six air-locks, and mounted on the caisson structure were six 2-ton electric derrick cranes, which handled the lock buckets for disposal of the excavated material from the working-chamber.

The Paper describes the compressor plant, the supply of compressed air to the working-chamber, the sinking operation, and the manner in which the caisson was arrested and plumbed when the cutting edge reached the required level.

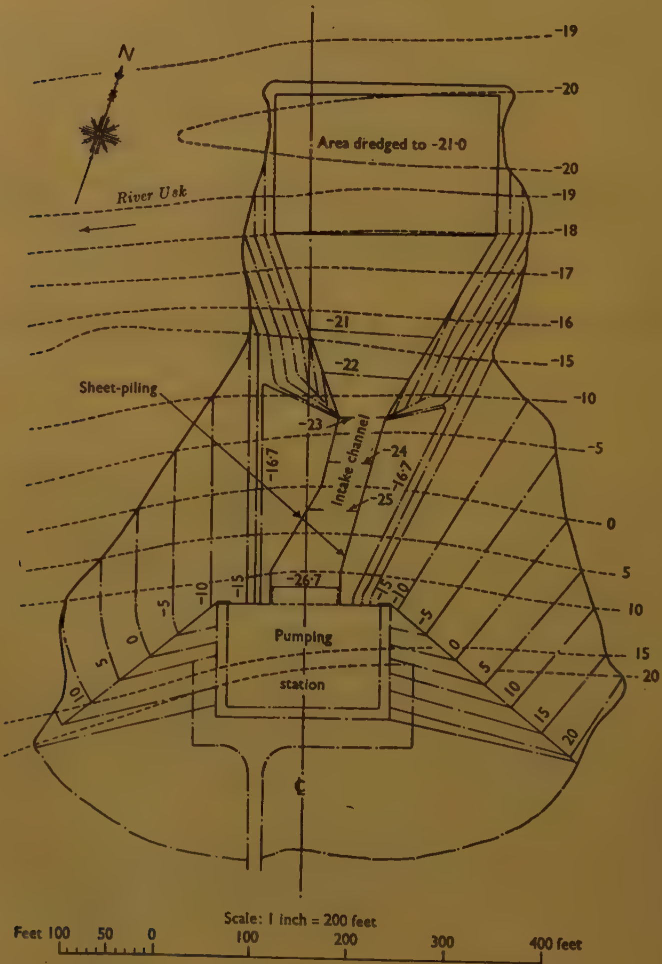
An Appendix to the Paper gives specimens of the various instructions, records, and log-books.

Introduction

The constructional work described in this Paper is for the circulating-water intake pump-house for the Uskmouth Generating Station, near Newport, Monmouthshire, now in course of construction for the British Electricity Authority.

The first part of this station will be equipped with six 60-megavolt-ampere turbo-alternators generating at 11·8 kilovolts, and twelve boilers, each having a coal consumption at full load of 18 tons per hour and an evaporative capacity of 360,000 lb of steam per hour at a pressure of 925 lb.

Fig. 1



SITE PLAN

per square inch. The maximum cooling-water requirement at full load will be 15,000,000 gallons per hour, or 18·6 tons per second.

At the site of the station, near the confluence of the River Usk and Severn estuary, the foreshore consists of very soft silty clay, slowly shelving

towards the river, and there is a large tidal range, leaving a narrow river channel in the centre at extreme low tides. To ensure a constant supply of circulating water at all tides, extensive initial dredging and canalization were necessary, and the pump-house had to be sunk deep into the river bed on the foreshore (*Fig. 1*).

The pump-house substructure was built and sunk as a caisson, measuring 164 feet by 110 feet in plan, and 80 feet in height, with foundation level at —51·75 O.D. (Newlyn). The maximum weight exceeded 42,000 tons, and the caisson is believed to be the largest in the world to be sunk under compressed air.

The general arrangement of the pump-house substructure is shown in Figs 2, Plate 1, from which it will be seen that it is divided into symmetrical halves by a central wall, each half comprising an intake chamber, screen and suction chambers, and four pump-wells. The invert level at the intake on the north face is at —26·7 O.D., and above this, in the central portion, accommodation for stores is provided. Along the east, south, and west faces of the substructure, twin ducts are provided; the upper one for electric cables and other services, and lower one for the circulating water, which is pumped to the condensers through culverts that branch out from the south-east corner of the pump-house. The twin ducts are supported, at their outer side, on reinforced-concrete piling, and on the pump-house substructure at the inner side. The pump-house substructure, above the level of the caisson shoe, is constructed in reinforced concrete. The pump-house building is steel-framed, with brick walls and pre-cast reinforced-concrete roofs, and is equipped with overhead cranes for servicing the pumps, screens, and penstocks.

Preliminary Considerations of Scheme

The design by the consulting engineers, L. G. Mouchel & Partners, on which tenders were invited, provided for two reinforced-concrete caissons to be built adjacent to each other on the foreshore, on a prepared hardcore bed with surrounding sheet-piling, and to be sunk under compressed air simultaneously, the one being kept a little in advance of the other. There was, however, an over-riding clause in the specification to the effect that, should the contractor wish to submit for the consideration of the engineers an alternative method of procedure, a statement should accompany the tender of the salient features of the alternative proposals.

The contractors, at the tender stage, after careful consideration of the report on the ground values, prepared by Soil Mechanics, Ltd, and of the tidal range and the general site conditions, formed the opinion that heavy reinforced-concrete caissons built on the foreshore might get out of control, particularly during the early stages of sinking, and accordingly decided to tender on a single caisson, with a fully plated steel shoe, completely erected on shore, rolled out and lowered into position on the foreshore, and sunk

in the usual manner. The tender on this alternative scheme was duly submitted, the consulting engineers and the British Electricity Authority approved the design, and the contract was awarded to Holloway Brothers (London) Ltd.

Design of Caisson-Shoe Steelwork

Consideration of the extent of the area to be excavated led to the decision to use six air-locks and, consequently, three working-chambers were adopted, with two air shafts to each. This arrangement allowed for the provision of two strong internal transverse bulkheads, accommodating spine trusses, into which were framed the roof girders over the working-chambers (see Figs 3, Plate 1). The side plating was made 20 feet 9 inches deep overall, with channel sections for the top flange and vertical stiffeners at the roof-girder spacings. The roof plating to the working-chamber was cambered to form vaults, which limited the weight of concrete which had to be placed in the early stages of sinking, and enabled the water and air pressures from below to be resisted by tensile stress only. The roof girders were connected together with ridge trusses, at the centre, and to the frames at the mitred ends of the vaults.

In the rolling-out operations, the ball tracks were placed immediately below the spine trusses (*Fig. 6*), and the whole structure was lowered from brackets (Figs 4, Plate 1) provided at the ends of these trusses. The form of framed construction adopted gave ample strength and rigidity in resisting the bending and torsional stresses set up during rolling-out and lowering, as well as during the early stages of sinking before the reinforced-concrete work was sufficiently far advanced to be adequate in itself.

As a matter of economy in steel, and in order to avoid the excessive amount of riveting and caulking required for watertight work, and since the erection (*Fig. 7*) was to take place ashore, under dry-land conditions, it was decided to make the shoe entirely in welded construction. The frames were fabricated in as large pieces as practicable, in the makers' works, and transported to site for erection and welding in place. The details were arranged to minimize positional site welding, and to provide downhand welding wherever possible. The vault-roof plating, $\frac{1}{4}$-inch thick, was sent to the site flat, inserted between the roof girders, and curved in place. The side plating was made in vertical strips, and welded to the outstanding toes of the vertical channel stiffeners. This was done to allow some spring in the plating and to avoid, so far as possible, internal stresses being set up. These plate joints were among the last to be welded.

Consideration was given to the sequence of erection (Figs 5, Plate 1) and welding to avoid distortion. The spine trusses were erected first (*Fig. 6*) and welded, followed by the cutting-edge details and the roof girders over the central portion. Then followed the erection of the outer and vault roof frames (*Fig. 7*), and the end frames with a number of the

vertical outside plates tacked in position. After a certain number of roof frames were erected, the roof plating was inserted and tack-welded at the centre, followed by full welding as the erection proceeded outwards. The vertical plating, as previously described, was the last part of the work to be fully welded. With this procedure, practically no distortion occurred and no cracking appeared in the welding.

The total weight of the steelwork in the caisson shoe was about 510 tons, whilst the weight of the welding rods used, which were of the Murex "Vodex" type, principally No. 6 and No. 4 gauge, was 6½ tons. The time taken in the erection and welding of the steelwork was 14 weeks.

Rolling out the Caisson

In order to reinforce the earth bund during the erection and rolling-out of the caisson shoe, and during the sinking of the caisson, a sheet-piled wall of Larssen No. 2 section was driven along the shore parallel to and immediately behind the site of the caisson. This sheet-piled wall had return ends and diaphragms, also of sheet-piling, at intervals of 70 feet along the wall. The front face had external walings, anchored by tie-rods, 2 inches in diameter, 40 feet long, at 10-foot-6-inch centres, to the back anchorages, which consisted of continuous pile sections laid horizontally in trench spanning between the diaphragms.

At right angles to the bank on the centre-lines of the spine trusses, in the area immediately behind the bund referred to above, two lines of 12-inch-by-12-inch timber piles were driven, to support the loads during the building and rolling-out operations, and behind this piled section the building- and rolling-tracks were mounted on cribbing, consisting of reinforced-concrete sleepers and timber longitudinal bearers and cross-bearers. Two bored tube piles were sunk into the marl in the correct position to support the lowering gear, at the shore and river ends of each of the two spine trusses. The inshore piles were used temporarily for supporting the bridging girders connecting the rolling-track where it passed over the bund.

The ground in front of the above-mentioned sheet-piled wall was excavated to a level bed, and the whole area on the site of the caisson was filled with hardcore, consisting of reject brickbats from a local brickworks, deposited and consolidated by end-tipping, and spread by bulldozers—thus squeezing the mud and silt forward as the work proceeded, and providing a uniform hard bed, on to which the caisson was eventually lowered. The roller tracks were carried over this filled area on prefabricated A-frame trestles (*Figs 8* and *9*), suitably braced in both lateral and longitudinal directions to resist the forces set up during the rolling operation.

The caisson shoe was built on roller carriages—eight to each spine truss—and special curved timber cradle supports were provided at each panel point of the trusses. The rolling gear was of the usual type, with 2⅜-inch-diameter steel balls rolling between the bosoms of two bullhead

rails lying on the flat; the rails on the carriages and track were welded to ½-inch-thick plates, dog-spiked to whole timbers.

Two hauling tackles, each of 20 tons capacity, were connected from the completed shoe to the trestle staging, with hand-operated haulage winches ashore, and safety check tackles were also provided. The structure, weighing 510 tons, was hauled into position (*Fig. 9*) ready for lowering, the operation being completed in 6½ hours.

Lowering the Caisson

At the four corners of the caisson site, guides were provided, consisting of sections of steel sheet-piling, driven in cruciform fashion in plan, and stiffened with 24-inch-by-7½-inch rolled-steel-joist sections as walings, to which timber corner fenders were attached, to prevent lateral movement of the caisson during the lowering. These cruciform guides were connected together outside the line of the four sides of the caisson with 1¼-inch-diameter wire ropes, tensioned up with straining screws.

The steel tube piles were extended after the rolling-out operation and the capping pieces were fitted to receive the lowering gear. These piles were 18 inches internal diameter (⅜-inch thickness of metal); they were sunk down to level and filled, partly with concrete and partly with dry sand.

The lowering brackets, which also controlled the positioning of the caisson, were threaded over these piles and welded to the caisson-shoe structure. The lowering gear consisted of a bridging girder supported on the top of the tube piles, upon which were mounted two 100-ton hydraulic jacks having an effective stroke of 12 inches. A second girder was supported on the rams of these jacks; so that the distance between the two girders could be adjusted by a clear 12 inches by operating the jacks. Both upper and lower girders were of plate-and-angle construction, each with two webs 2½ inches apart, bored on the centre-line to receive 5-inch-diameter steel pins. Threaded in the spaces between the webs of the girders were long steel tension links, 16 inches wide by 2 inches thick, with a series of 5-inch-diameter pin holes, at 12 inches pitch, corresponding to the jack stroke. The links had bolted connexions to the lowering brackets.

To carry the weight of the structure, the eight jack rams were pumped up until the four holes in the links registered with the holes in the upper girders, and the 5-inch pins were inserted when further movement upwards of the jack arm lifted the structure. This was done as a preliminary to the lowering operation, and the screw collars with which the jack rams were provided were screwed down tight. At this stage, the supporting A-frames and rolling tracks were removed from under the caisson. The caisson structure, now ready for lowering to its prepared bed, was lifted slightly, and the screw collars were run back, and the jack rams lowered by a complete stroke, until the holes in the lower girders coincided with the holes in the links, when the pins were inserted and the upper pins removed.

Fig. 6

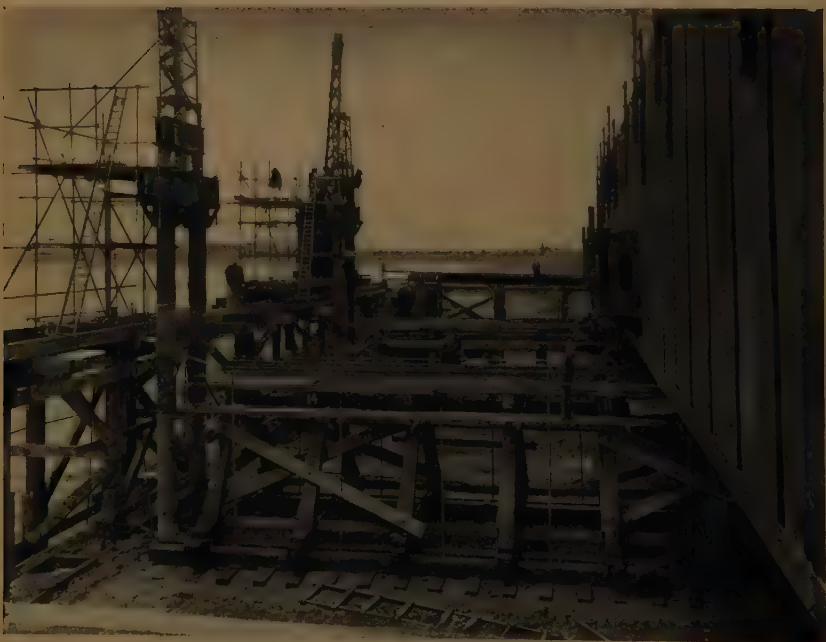
SPINE TRUSS OF CAISSON SHOE BEING ERECTED ON ROLLING-OUT BERTH

Fig. 7

Fig 8

AERIAL VIEW OF CAISSON SHOE PRIOR TO ROLLING-OUT

Fig. 9



VIEW DURING ROLLING-OUT OF CAISSON SHOE

Fig. 10

View Showing Interior of Compressor House

Fig. 11

Fig. 12

General View during Construction

Fig. 16

View inside one Working-Chamber during Sinking

This was done simultaneously on all jacks, and the structure lowered by 1 foot. The process was then repeated, foot by foot, and continued until the structure became waterborne, when all pins were removed. Concrete was then placed in the caisson haunches until sufficient weight had been added to overcome flotation at high tide.

After lowering the caisson shoe, the lowering gear was dismantled, the guide brackets burned off, the tops of the tube piles cut off, and, before sinking proceeded very far, the corner guide timbers were removed.

Description of Plant for the Caisson

After the erection yard (where the caisson had been built) was cleared of all temporary structures, a 10-ton electric derrick crane, with a jib 120 feet long, was erected ashore on a braced steel gabbard, with sill level at +51·00. The legs of the gabbard were steel-tube piles sunk to level —49·00. As previously explained, six air shafts and air-locks were provided, and were erected by this 10-ton crane. For serving the air-locks, six 2-ton fast-running electric derrick cranes, with 45-foot jibs, on steel gabbards, were erected on the caisson, one to each air-lock. These cranes were arranged so that the muck from the air-locks could be loaded on to decauville track wagons which ran on timber stagings along the sides and river face of the caisson.

The six air-locks were of the type described by the Authors in an earlier Paper.[1] The materials shafts to the air-locks were used almost entirely for excavation, separate arrangements being made for pumping the concrete into the working chamber. All the concrete work in the pump-house substructure was put in by concrete-pumps supplied by a fully mechanized batching plant.

The concreting plant consisted of a weight-batching plant with drag scraper and elevator for aggregates and sand delivering into a three-compartment top bunker. It was fitted with an automatic elevator for handling the bulk cement into an elevated silo of 60 tons capacity. The concrete mixer, of 1 cubic yard capacity, was placed above the two concrete-pumps and fed into the hoppers of each, as required, by means of a breeches chute. One 6-inch " Blaw Knox " concrete-pump and one 4-inch " Pumpcrete " pump were employed, the pipe-lines delivering concrete direct into place. At a later stage in the work, the 6-inch pump was replaced by another 4-inch pump.

Description of the Compressor Plant

The compressor plant (*Fig. 10*) consisted of four Alley & McLellan low-pressure compressors, each with a capacity of 300 cubic feet of free air per minute, two Broom & Wade 300-cubic-feet-per-minute compressors—all

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[1] W. Storey Wilson and F. W. Sully, " Compressed-Air Caisson Foundations." Works Construction Paper No. 13, Instn Civ. Engrs, 1949.

designed for a working up to 50 lb. per square inch—and two Broom & Wade compressors of similar capacity, but working up to 100 lb. per square inch. All these machines had electric drive, with the exception of two of the Alley & McLellan compressors, which were steam-driven, fed by two Robey oil-fired boilers, having an evaporation capacity of 2,100 lb. of steam per hour at a gauge pressure of 120 lb. per square inch. Normally, electric power was used, but the boilers were kept under steam at all times as a standby in case of a breakdown of the electric supply. The low-pressure machines were fitted with after-coolers, with both steam and electrically driven pumps, supplying cooling water at 40 lb. per square inch circulating through a 12,000-gallon storage tank.

The low-pressure compressors delivered compressed air into a 6-inch bus main with branches to three air receivers alongside the compressor house. From each of these receivers, a 4-inch main led through a "Vokes" air filter to the river bank alongside the caisson, and inter-connecting bus pipes and valves were provided at the compressor house and river ends to allow for any main to feed any part of the caisson. Air was delivered into the caisson through 3-inch flexible pipes to automatic non-return valves, situated one above each of the three working-chamber roofs, thence inside the working-chamber through 3-inch pipes with 2-inch branches which conveyed the fresh air to the corners of the caisson, where non-return flaps were fitted to the outlets of the 2-inch pipes. Air was exhausted through the adjustable safety valves of the air-locks. A Budenberg automatic pressure recorder was fitted to each air-lock. High-pressure air for tools, and fresh-water supply, were supplied to each working-chamber.

Comparative costs of steam and electricity were kept, from which it was found that the cost of fuel oil for steam raising was about four times greater than the cost of electricity. Hence, although the boilers were kept with a full head of steam ready at all times for an emergency, they were not used normally.

Such an emergency occurred on four occasions; three times when it became necessary to cut off the electrical power supply, and the fourth time when, by accident, the main electrical feeder cable was fouled elsewhere on the site. In all cases, the switch-over was done smoothly, without loss of pressure, owing to the very effective non-return valves fitted to the air supply at the caisson end, and the rapidity with which the steam-driven compressors were brought into operation.

Air Supply to Caisson

The amount of air was determined by the requirements of the Factory Act Draft Regulations, which amount was always more than that required to maintain pressure. When the cutting edge was sealed in clay, the surplus air was allowed to escape through safety valves on the air-locks, adjusted to the varying pre-determined pressures.

The amount laid down by the Regulations is 10 cubic feet of air per man per minute, at the pressure of the working-chamber. With a maximum force of forty men at a pressure of 27 lb. per square inch, 1,120 cubic feet of free air per minute was required, which was within the capacity of the four sets driven by electricity, without calling on the auxiliary electrical or steam supply. On the occasions when reliance had to be placed on the steam-driven compressors alone, with all men out and doors closed, about 125 cubic feet of free air per minute was sufficient to maintain the working-chamber pressure over an extended period.

It was found that about 200 to 300 cubic feet per minute of extra air were required when "Snorers" were in use.

Caisson Sinking Operations

The concrete work in the haunches of the caisson shoe had to be carried out carefully and expeditiously, to prevent undue distortions and to avoid the caisson becoming buoyant during the spring tides following the lowering of the caisson to the prepared level at +12·00 O.D. *Figs 11* and *12* are two general views of the caisson during construction.

On the 3rd August, 1950, when the cutting-edge had sunk to level +11·50, concreting was commenced and continued day and night, in shallow lifts, until 1st September, 1950, when about 6,572 cubic yards had been placed and the cutting edge reached level +4·00. During that period, the distortion of the caisson shoe and the level of the cutting edge were checked almost hourly, and the placing of concrete was directed to keep the caisson on an even keel and to prevent distortion of the shoe structure. During this stage of sinking no effective attempt was made to excavate in the working-chambers, because the soil had flowed up to the caisson roof. Compressed air was put on with the cutting-edge level at +4·00, on the 10th September, 1950, and excavation started slowly at first, at the bottom of the air shafts, but, as sinking proceeded and stability was obtained, the excavation at the bottom of the shafts was extended, and in due course the working-chambers were cleared of material down to the cutting edge.

In the meantime, further concrete was placed, to a carefully arranged plan to suit the programme, the outer walls of the pump-house substructure being constructed in advance of the inner walls, to support the skin plating of the shoe against the water pressure from outside, and to provide a coffer-dam for the interior work. The work was carried out in sections in plan, arranged to give the most efficient cycle of operations of reinforcement fixing, formwork setting, and concrete placing, and at the same time to give the necessary weight distribution to correct any tendency for the caisson to tilt, as determined by periodic measurements. Fig. 13, Plate 2, is the sinking record chart, on a time basis, showing the predicted high and low tide levels, the progress in building the substructure, and the cutting-edge level. The actual tide levels in this area are rather erratic, and

an ample margin was required in the construction level of the concrete work to eliminate the risk of flooding at high water.

It was anticipated that, during sinking, the caisson would drift towards the river, and for this reason it was set 1 foot shorewards of its designed position. It was impossible to predetermine the amount of drift, but the engineers agreed that, whilst the founded level and plumbness of the caisson were of vital importance, the final position—within reasonable limits—was of no consequence, and, accordingly, whilst every endeavour was made to keep the caisson plumb during the period of sinking, no attempt was made to arrest the drift; it is doubtful if, under the conditions that existed, any such attempt would have been successful.

In the early stages of sinking, the soil squeezing out under the cutting edge, under the influence of the pressure exerted by the caisson roof, met little resistance, on the river side, and just flowed out in a wave towards the channel; whereas the soil squeezing out on the in-shore side created a pressure between the caisson and the protective sheet-piled wall, causing an outward thrust and forward movement of the caisson. This movement continued fairly uniformly until the cutting edge reached level —10·00, when the caisson had moved 4 feet forward from its original position. By this time, conditions had become stable, with the working-chambers cleared of material, and from then onwards the drift, though still persisting, was much less, and the caisson, when founded, was, on an average, 4 feet 9 inches forward of its original position, or 3 feet 9 inches from its designed position.

Figs 14, Plate 2, show the relative positions of the caisson during the sinking operation. The isometric views show the relative movements, the distortion of the cutting edge to exaggerated scale, and the drift riverwards referred to above.

As the caisson sank through the clay in the early stages of sinking, a shallow depression was formed around its perimeter, owing to the adhesion of the clay to the sides of the caisson, which depression did not drain at low tide, leaving a "puddle." During sinking through the clay, the air pressure applied represented the hydrostatic head of the "puddle," when the tide was below that level.

From the result of the boreholes, it was known that a sand stratum occurred, at about level —26·00, and gravel at about —40·00. The sand and gravel strata were known to be water-bearing, showing a damped tidal range.

It was not known if or where these strata ran out in the channel bed, and, consequently, as the caisson sank and the air pressure increased to balance the "puddle" head, there was a danger of a "blow" in the river bed, at low tide, if the sand and gravel strata did outcrop in the channel.

To guard against this possibility, the low-tide air pressure, after the cutting edge reached level —22·00, was reduced slightly, permitting water to enter the caisson and just drown the cutting edge. The pressure

necessary for this condition was found to be roughly midway between the "puddle" pressure and the true low-tide pressure. The pressure established in this manner was then increased by about 1 lb. per square inch, to keep the work dry, and, making allowance for level of the cutting edge, this low-tide pressure was maintained throughout the remainder of the sinking.

The sand was extremely fine grey sand, oxidizing in air to pale yellow, similar to dune sand. In its natural state it was porous, but it was found very difficult to blow back seepage water by air pressure. During normal working, the water was usually 6 to 9 inches below the surface, and as excavation proceeded this dryness was maintained without increasing the low-tide air pressure mentioned above, even when the excavation was at times as much as 18 inches below the level of the cutting edge. After "blowing down" in sand on one occasion about 18 inches of water entered the caisson under the cutting edge, but comparatively little sand was drawn in. It was extremely difficult, however, to get rid of the water with air pressure, and it was more convenient to blow it out through "snorer pipes."

As the caisson approached the gravel stratum, the air pressure established as described above was insufficient to keep the working chamber dry and a certain amount of water entered, but was easily dealt with by "snorer pipes." At this time also a time lag, between actual high tide and the apparent high tide in the caisson, of about half-an-hour, was recorded, and, for safety, the high-tide pressure was maintained for 1 hour after high tide. In this way the caisson was kept reasonably dry through the high-water period, but when in the gravel an appreciable amount of air was lost under the cutting-edge, blow holes appeared in the river bed, and, at certain high tides, some water entered the caisson. The air pressure could not be raised to counteract this; air was passing straight out under the cutting edge, at the high end, because it was not quite level. However, this seepage water was led away in drainage channels and drawn off by "snorers" without difficulty.

During "blow downs" in the gravel at low tide, only a small quantity of water entered the caisson, and, since the blow holes in the river bed did not diminish appreciably, it can be inferred that the whole of the gravel layer was acting as an air-receiver.

In the marl, the caisson sealed itself as it had done in the soft clay at the higher levels, except here and there where blowing had occurred through the gravel.

Referring to Fig. 15, Plate 2, the following data are plotted against the appropriate level of the cutting edge during the sinking operation:—

(*a*) The actual weight of caisson during sinking.
(*b*) Buoyancy curve for H.W.L. +23·6.
(*c*) Buoyancy curve for W.L. +10·00 ("puddle head").
(*d*) Buoyancy curve for L.W.L. —18·5.
(*e*) Estimated safe load on soil.
(*f*) Estimated failure load on soil.

Curves (*e*) and (*f*) were obtained from results of a typical boring taken near the west side of the caisson during the sinking operation by the vane test method. The safe bearing pressure intensity was taken as:

$$P_1 = 2S + \rho h(\gamma - 1)$$

where S denotes maximum shear value from the vane test, h the height of ground above the plane required, ρ the weight of water per cubic foot, and γ the specific gravity of the soil. The critical value where soil failure could be expected to occur was assumed to be, similarly:

$$P_2 = 6S + \rho h(\gamma - 1)$$

The caisson attained a measure of stability very nearly where the curves (*a*) and (*e*) intersected; that is, about level —16·00, when the outward drift of the caisson had practically ceased.

Skin Friction

Only one occasion presented itself for testing the value of skin friction. That occurred when blowing-down with the cutting edge at level —47·00 with ground level +12·00. The cutting edge was clear all round; the pressure was reduced slowly and movement started at a gauge pressure of 18 lb. per square inch. This gave a value for skin friction of 10·3 cwt per square foot. It will be noted from Fig. 13, Plate 2, that the caisson had previously not moved for a week.

Excavation

Preliminary investigations were made as to the advisability of pumping out the excavation from inside the working-chamber. That was not proceeded with when it was found that the ratio of water to clay to make a pumpable mixture was about 10 : 1. The disposal of the material would have caused considerable difficulty, and, since the construction work above, rather than excavation, was the controlling time factor, it was decided to use normal digging methods.

Drag scrapers were installed in one of the working-chambers. They consisted of air motors mounted on the working-chamber roof, near the air shaft, with rope attachment fitting to the haunch steelwork. The scraper, working towards the shaft, dumped the spoil into the air-lock buckets, but this method was abandoned as better progress was made with hand digging, the buckets being manipulated into position through snatch blocks.

The ideal excavation force consisted of the following:—

One foreman in charge of the shift.
One charge-hand.
Six gangs, one per shaft, each consisting of:
- 1 leading hand,
- 1 inside lock-keeper, and
- 3 or 4 labourers.

The total was thirty-two to thirty-eight men, of whom twenty-four to thirty would be actually digging.

Figs 17

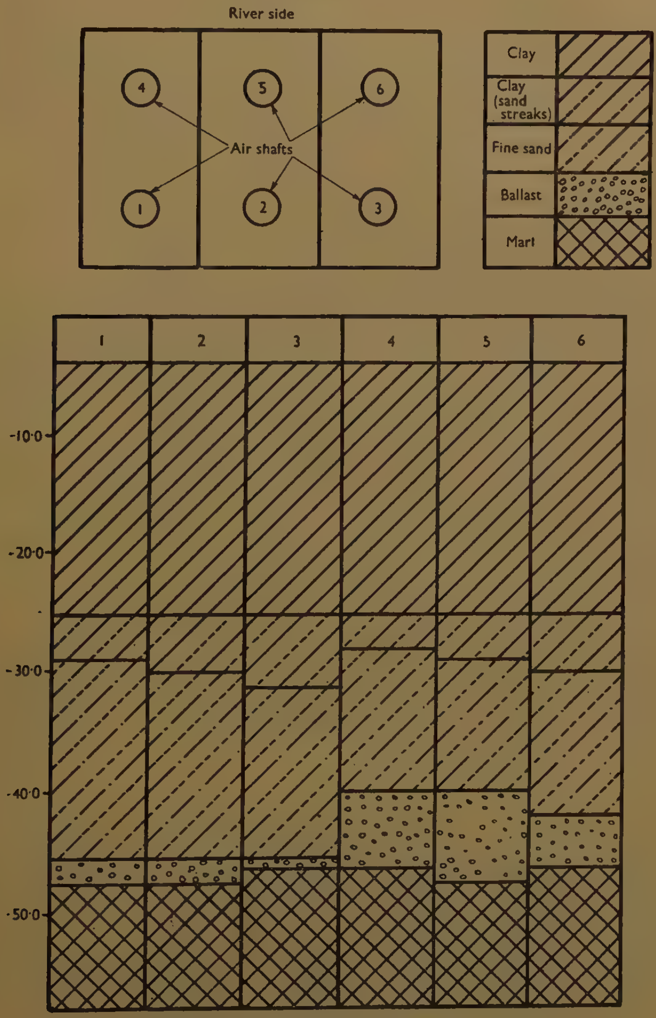


DIAGRAM SHOWING MATERIAL EXCAVATED

In practice, only five or sometimes four gangs were employed, since it was generally necessary to suspend excavation work in one or two shafts, every shift, to correct changes in the level of the caisson.

An effective bonus system based on output was operated throughout. It was found that the nearer the bonus payment could be based on individual effort the more effective it became. Each gang was bonused on its own daily output. The maximum output recorded from one shaft averaged eight skips per hour when in sand with four men digging. A checker was fully employed in recording the skips raised from each shaft.

The maximum excavation in sand recorded in a 24-hour period, with all six shifts working, was 745 skips, representing 466 cubic yards in the solid. The average rate of excavation in 24 hours under normal working conditions was more nearly 220 cubic yards measured in the solid.

Sealing the Working-Chamber

It was essential to sink the caisson as accurately to the prescribed level, and as truly plumb, as possible, since all penstock openings and other details had been constructed while the caisson was in the process of sinking. The sinking operations were suspended when the cutting edge was about 3 feet from its final level, and a reinforced-concrete slab, 10 feet wide and 3 feet thick, was built under, and for almost the whole length of, each of the two bulkheads. The upper surface of the slab was screeded to the correct profile and level of the underside of the bulkheads. On 6th June, 1951, the caisson was very gently blown down until it came to rest on these two slabs with an average error in level of $\frac{1}{8}$ inch; in verticality of $\frac{3}{8}$ inch in 164 feet; and transversely, of $\frac{1}{4}$ inch in 110 feet.

The concrete in the working-chambers was pumped in through four auxiliary pipe air-locks, through which the pump mains passed. The main was brought to a horizontal line at its end and fitted with a flap.

On completion of the concrete emplacement, the working-chamber was grouted up, under air pressure, through the air shafts and, 3 days later, the air pressure was taken off.

Safety and Welfare

Every effort was made to carry out all operations under compressed air in strict accordance with the undermentioned existing and proposed regulations of the Ministry of Labour:

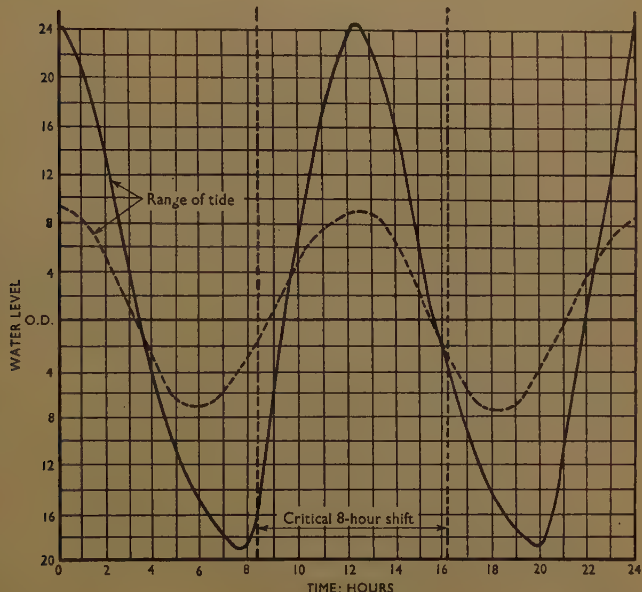
> The Factory Act 1937—Work of Engineering Construction Part VII—Work in Compressed Air, Provisional Draft dated June 1948; amended later by a further draft revision dated 1951.

These regulations covered all requirements and were satisfactory, except in regard to the rate of decompression after recompression in the medical air-lock. At Uskmouth, the pressure in the medical lock was built up until the patient was relieved of all pain, which usually happened at a pressure lower than that of the working-chamber, and was then reduced

at a constant rate of 5 minutes per pound per square inch. A copy of the notice posted in the medical lock is given in the Appendix, Form A.

By adhering to the method laid down in the draft Regulations for calculating the "working" pressure, the time for decompression at the end

Fig. 18



Cutting-edge level	Largest tide range: Level at mid shift	Largest tide range: Level at end of shift	Largest tide range: Mean head	Largest tide range: "Working" pressure	Smallest tide range: Level at mid shift	Smallest tide range: Level at end of shift	Smallest tide range: Mean head	Smallest tide range: "Working" pressure
0	24·3	—	12·1	5¼	9·25	—	4·6	2
—20	44·3	15·5	29·9	13	29·25	16·25	22·8	10
—33	57·3	28·5	42·9	18½	42·25	29·25	35·8	15½
—39	63·3	34·5	48·9	21	48·3	35·0	41·7	18
—52	76·3	47·5	61·9	27	61·25	61·25	54·8	24

ESTIMATION OF "WORKING" PRESSURE TO BE USED FOR TIME OF DECOMPRESSING MEN IN THE AIR-LOCK AT END OF SHIFT

of the shift was reduced below what it would have been had maximum tidal pressures been taken as the working pressures. This is shown in *Fig. 18*, which indicates the working pressure under different tidal conditions. Auxiliary air-locks similar to the medical lock were provided for

"decanting" the men, but were not used, since all normal decompression was carried out in the air-locks to the working-chambers.

When the "working" pressure exceeded 18 lb. per square inch, the following procedure for decompression was carried out:—

Towards the end of each shift, the first-aid attendant filled in a record card (Form B in the Appendix) and, when this had been checked and signed by the shift engineer, the information was passed to the lock-keepers, who were responsible for decompressing the shift in accordance with that information.

Each lock-keeper had his own book, in which he recorded every person entering and leaving his air-lock. This information was passed to the first-aid attendant, who recorded it in a main log book. Separate log books were kept for day and night shifts. A specimen of this main log book is given in Form C in the Appendix.

Every person, before being engaged for compressed-air work, was medically examined and, if accepted, was again examined at the stipulated intervals during the period of employment on compressed-air work. A sample medical card is given in Form D in the Appendix. Every man engaged for compressed-air work was given a printed copy of "Notice to Compressed Air Workers"—Form E in the Appendix.

Throughout the period of compressed-air working, tests were carried out on the air in the working-chamber, at least once every shift, for:—

(*a*)	Carbon monoxide ..	CO
(*b*)	Hydrogen sulphide ..	H_2S
(*c*)	Methane	CH_4
(*d*)	Carbon dioxide	CO_2
(*e*)	Oxygen deficiency	

A record for these tests was kept and signed by the shift engineer. A sample page (Form F) is given in the Appendix. Except for the presence of carbon monoxide, all tests were negative. On the 31st January, 1951, during a thick fog, the test for carbon monoxide gave a dangerous indication, and all men were withdrawn from the caisson. It was suspected that fumes from open braziers near the compressor house were blanketed down and drawn into the intakes. Again, at a later date, a certain amount of oxy-propane burning was undertaken in the working-chambers, and tests were made continuously during this operation. It was found that, despite the fact that hoods connected to a "snorer" pipe were held immediately over the work, carbon monoxide was present to a dangerous extent, and the men had to be withdrawn from time to time until the gas cleared.

The test for carbon monoxide provides for a sample of the air being passed through a glass tube containing silica gel at each end and palladium sulphite crystals in the centre, about 2 inches long. A lethal dose of carbon monoxide is indicated by a black discolouration of about $\frac{1}{4}$ inch in the palladium sulphite.

Caisson Sickness

In every case of a man being admitted to the medical air-lock suffering from "bends" a supplementary accident report was recorded—Form G in the Appendix.

The following is a summary of the cases of caisson sickness treated :—

52 men lost no time.
12 men lost one shift.
1 man lost two shifts.
1 man lost five shifts.

Only one man was taken off sinking work entirely because of disability. He had a moderate attack of bends after his first day in air and was only free from pain after recompression for a second time.

Altogether, 9,961 man-shifts were worked under air pressure, 3,911 of them at pressures higher than 18 lb. per square inch. The total number of cases treated for bends was 66, which, against the 3,911 man-shifts, represents 1·7 per cent.

Susceptibility to bends was indicated by the following data :—

1	man	was	treated	on	7	different	occasions.
1	,,	,,	,,	,,	6	,,	,,
2	men	were	,,	,,	4	,,	,,
2	,,	,,	,,	,,	3	,,	,,
11	,,	,,	,,	,,	2	,,	,,
17	,,	,,	,,	,,	1	occasion	

No case could be termed severe. All cases were able to walk to the medical lock without more than leaning on the attendant.

There were, unfortunately, two fatal accidents in the course of the work, neither of which was due to compressed-air working.

Acknowledgements

To L. G. Mouchel & Partners Ltd, the Authors extend their thanks and appreciation of the help given by the Consulting Engineers to the contractors in carrying out the work and in modifying the design of the permanent work where desirable, to facilitate construction.

The structural steelwork was fabricated by The Cleveland Bridge & Engineering Co. Ltd, of Darlington.

The assistance given by H.M. Engineering Inspector of Factories of the Ministry of Labour and National Service, in finalizing the welfare arrangements, in so far as these concerned compressed-air work, was of the greatest help to the contractors.

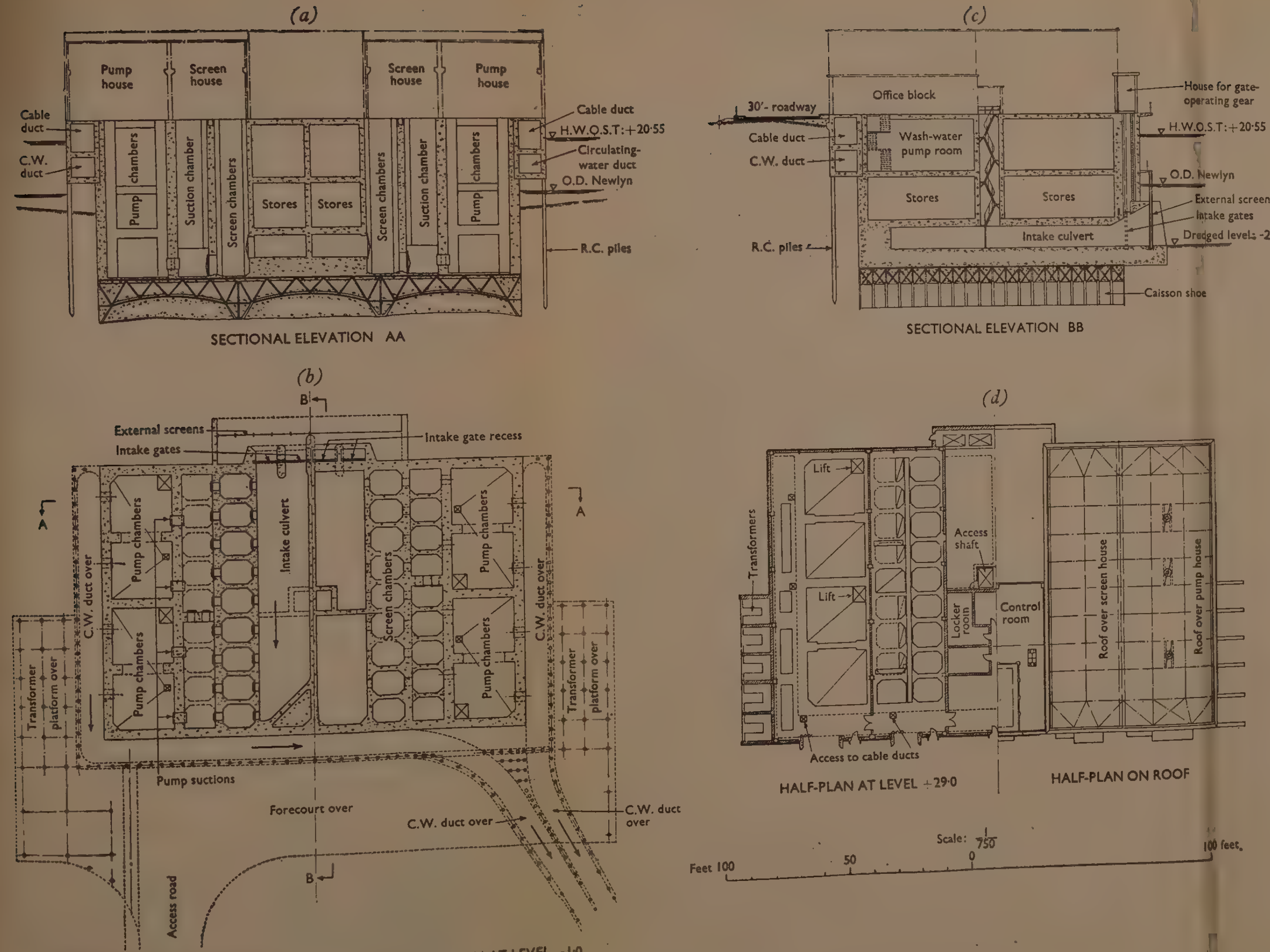
Finally, the Authors beg to thank the British Electricity Authority and the Directors of Holloway Bros (London), Ltd for permission to

publish information contained in the Paper, and the latter also for defraying the cost of the model and film of the work.

The Paper is accompanied by eight photographs and ten sheets of drawings and diagrams, from which the half-tone page plates, folding Plates 1 and 2, and the Figures in the text have been prepared; and by the following Appendix.

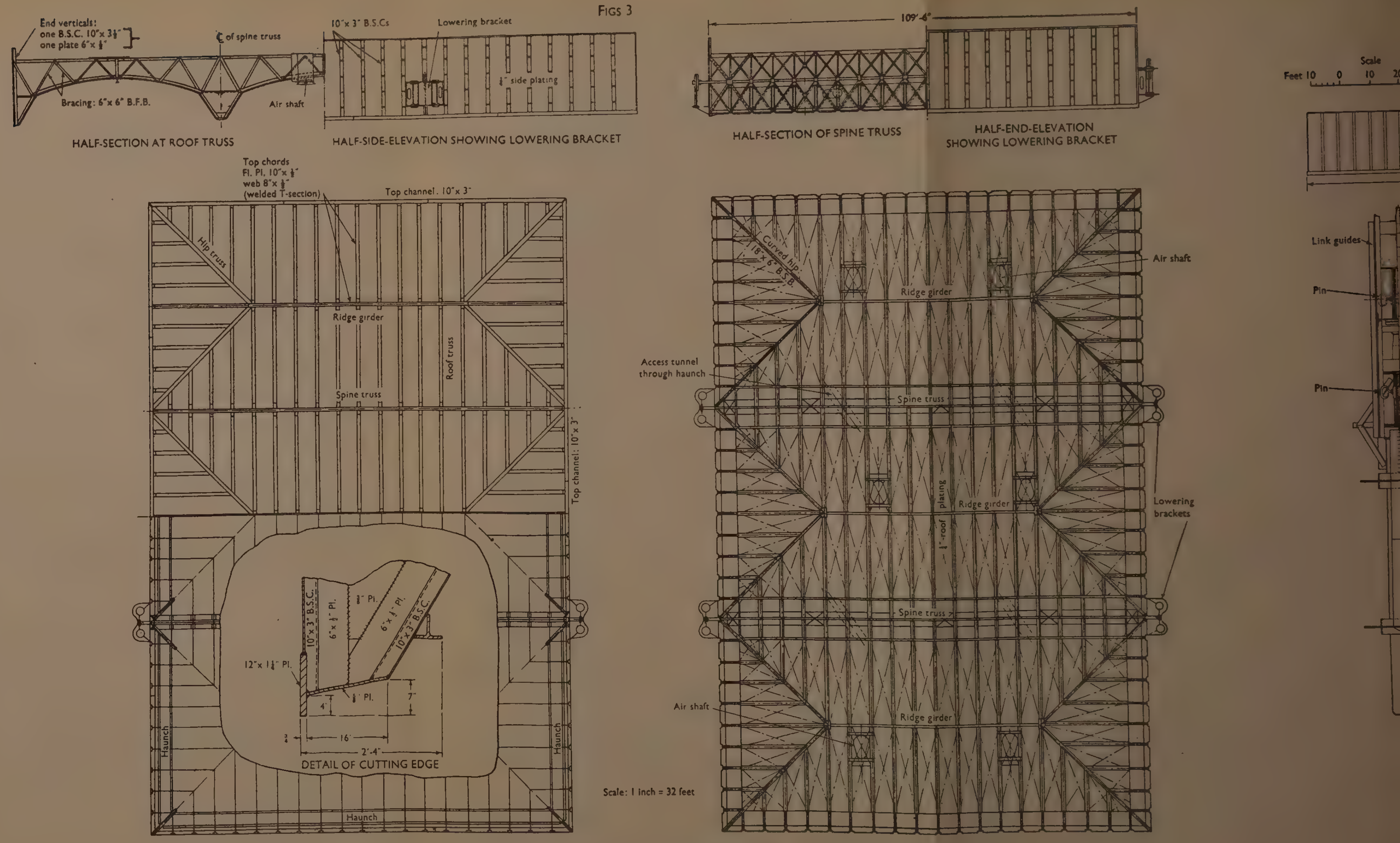
THE CONSTRUCTION OF THE CAISSON FORMING THE FOUNDATION TO THE CIRCULATING-

Figs 2



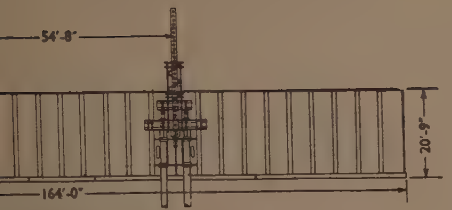
GENERAL ARRANGEMENT OF PUMPING STATION

Figs 3

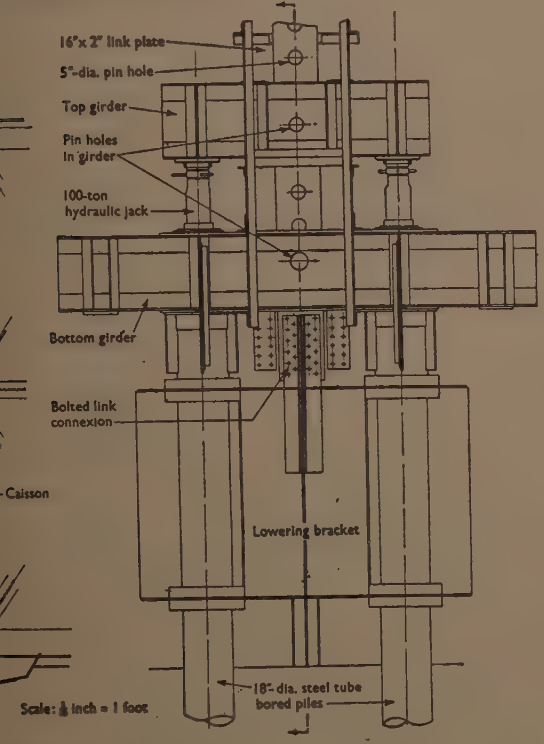


DETAILS OF CAISSON-SHOE STEELWORK

Figs 4

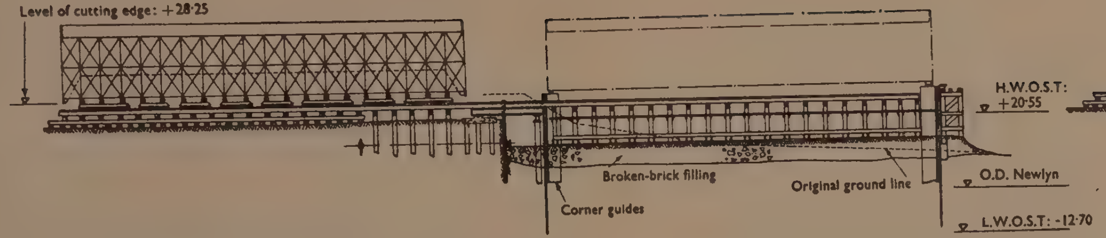


ON SHOWING LOWERING GEAR



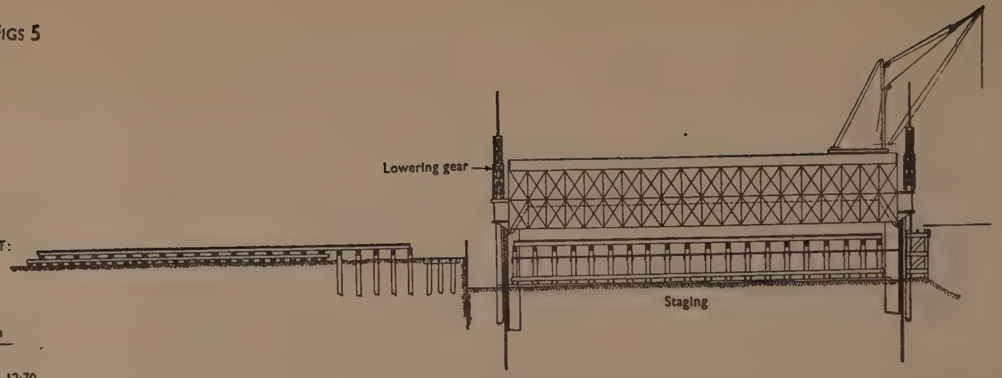
ANGEMENT OF LOWERING GEAR

Figs 5



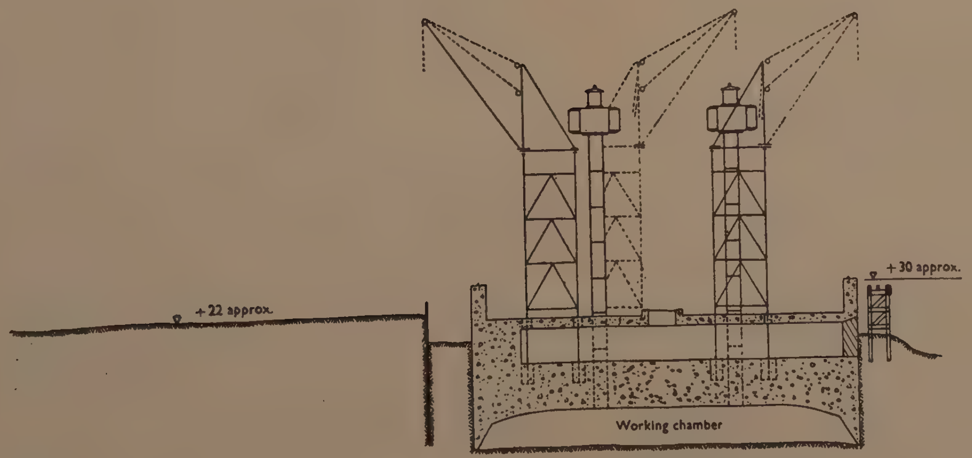
STAGE 1

CAISSON SHOE IN ERECTION POSITION ON SHORE



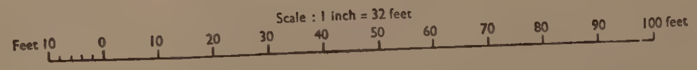
STAGE 2

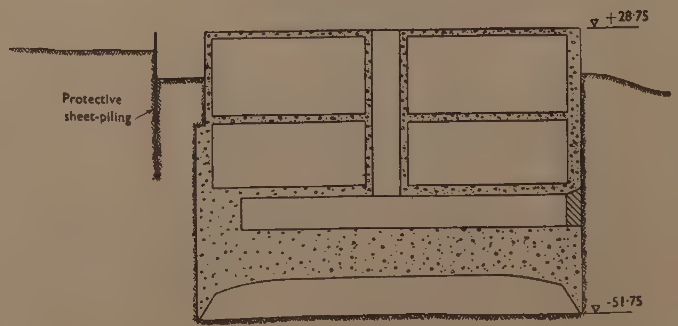
CAISSON SHOE SUPPORTED ON HYDRAULIC JACKS PRIOR TO REMOVAL OF STAGING AND SUBSEQUENT LOWERING



STAGE 3

CAISSON DURING SINKING WITH TOWER DERRICKS AND AIR LOCKS IN POSITION





STAGE 4

CAISSON AFTER SINKING TO FINAL LEVEL

DIAGRAMS SHOWING STAGES OF CONSTRUCTION OF CAISSON

W. STOREY WILSON and F. W. SULLY

APPENDIX

Rules of procedure for employees working in compressed air and records system adopted for the Pump-House Caisson at Uskmouth Generating Station.

FORM A

NOTICE

MEDICAL LOCK

THE PATIENT MUST NOT TOUCH THE CONTROLS.
THE PRESSURE WILL BE BUILT UP SLOWLY.
THE PATIENT MUST INFORM THE OPERATOR AS SOON AS THE PAINS DISAPPEAR.
THE PRESSURE WILL BE DROPPED SLOWLY—ABOUT 5 MINUTES FOR EACH POUND OF AIR.
DURING THIS PERIOD THE PATIENT SHOULD SWING HIS ARMS AND MOVE ABOUT.
DURING DECOMPRESSION THE INLET VALVE WILL BE KEPT SLIGHTLY OPEN TO PROVIDE FRESH AIR.

HOLLOWAY BROS (LONDON), LTD.

Notice Posted in Medical Lock

FORM D

Name : .... Dr's Name : ....
Address : .... Address : ....

Date of Exam.	Certificate No. and Expiry Date	Doctor's Remarks	Doctor's Signature	Date of illness	
				Off	Return

Medical Case Form

FORM F

Date	Time	CO.	H_2S	CH_4	CO_2	Oxygen deficiency	Engineer's signature

Record Book for Indication of Presence of Poisonous Gases

FORM B

DATE	SHIFT	PRESSURE AT MID-SHIFT	PRESSURE AT END SHIFT	WORKING PRESSURE	PRESSURE DROP TO IN FIRST 2 MINS.	RATE OF DROP FOR REMAINDER (MINS. PER LB.)	TOTAL TIME	ENGINEER'S SIGNATURE

FORM TO BE FILLED IN BY FIRST-AID ATTENDANT AND SIGNED BY THE SHIFT ENGINEER AND INSTRUCTIONS PASSED TO LOCK-KEEPER

FORM C

Date.........19..

NO.	NAME	ENTERED		MID-SHIFT		LEFT		Working Pressure	REQUIRED TIME		TIME TAKEN	
		Time	Pres.	Time	Pres.	Time	Pres.		Pressure after 2 mins	Time from 2 mins to zero pressure	Pressure after 2 mins	Time from 2 mins to zero pressure

FORM OF LOG BOOK KEPT BY THE FIRST-AID ATTENDANT

FORM E

Notice to Compressed Air Workers

General

No unauthorised person may enter any airlock or chamber under pressure, and no person not previously experienced in compressed air work may enter except under the escort of someone who has previously worked under air pressure.

No person may enter any airlock or chamber under pressure unless he has been passed fit by the doctor appointed by the firm and been duly entered as fit in the firm's site register.

After an absence from work due to sickness or accident lasting more than three days, re-examination by the appointed doctor before re-commencing work is compulsory.

When working pressures exceed 18 lb. per square inch gauge pressure, every caisson worker must be medically examined by the appointed doctor and passed fit at intervals of one month.

It is forbidden to consume alcohol while in compressed air.

It may be necessary to prohibit smoking while working in the caisson at times when it is feared that inflammable gases may be encountered during sinking. Instruction to this effect will be posted on the airlocks when necessary and must be strictly obeyed.

Every caisson worker will be issued with a numbered disk giving the information required by the police or doctors in case a man should be suddenly overcome by caisson sickness. This disk should always be worn both at and away from work.

Should a caisson worker be attacked by pains in the knees or elbows or other symptoms of caisson sickness, he is advised to make his way back to the site as quickly as possible and report to the ambulance man, who will immediately arrange his treatment in the medical lock.

Precautions on Entering Caisson

Caisson workers will enter the lock in groups, at least one of whom has had previous experience in compressed air work. One man will be appointed out of each group to control the pressure valves while passing through the man-lock. Any man should warn his leader immediately if he feels pains in the ears or head. The leader will then slow down or stop the entry of air until the pain has eased. Any worker who regularly suffers pain in the ears or head when entering is unsuitable for this type of work.

It is not advisable to go into compressed air when suffering from a cold in the head.

Precautions re Leaving Compressed Air

Strict Regulations exist regarding the time of decompression when leaving a compressed air chamber. In order that these regulations can be properly carried out, the decompression of parties of men in the man-locks will be effected not by the men themselves but by the outside lock-keeper who will receive his instructions from the office as to the speed of decompression.

No attempt must be made to take control away from the outside lock-keeper, but in case of discomfort or illness in the lock, contact can be made with the lock-keeper by means of the speaking tube provided.

After leaving the lock the men should proceed directly to their shelter where they will be served with hot coffee, to be drunk immediately.

FORM G

CASE NO :

SUPPLEMENTARY ACCIDENT REPORT

For use in the case of Caisson Sinkers at Uskmouth

Date of treatment ....

Man's Name ....

Man's No.............

Date started work in compressed air ....

Date of last attack of caisson sickness ....

Date of last medical examination ....

Symptoms ....

WORKING CONDITIONS		TREATMENT
Last shift started	...............date ....time	Time pains first felt .... Returned to site from ....
ended	...............date ....time	Brought to site by ....
Pressure beginning of shift	.... lb.	Time arrived at site ....
Pressure end of shift	.... lb.	Examined on site by ....
Pressure middle of shift	.... lb.	Time entered Medical Lock............
" Working Pressure "	.... lb.	Pressure when pains disappear .... lb.
		Time at which pains disappear ....
		Decompression rate (every 2 lb.) :—

Time taken to decompress at end of shift :

(*a*) Reduced to .... lb. in .... min.

(*b*) Reduced from .... lb. in .... min.

Time	Pressure
....	....
....	....
....	....
....	....
....	....

REPORT FORM FOR MAN ENTERING THE MEDICAL LOCK SUFFERING FROM " BENDS "

Discussion

The Authors introduced the Paper with the aid of a film and a sectional model of the caisson.

Mr H. J. B. Harding observed that the decision to use caissons for the pump-house at Uskmouth had been taken by the late Dr Mautner of Messrs Mouchel & Partners, a caisson sinker of international fame until he had been evicted by the Nazis, which, as usually happened in cases of persecution, had proved to be to Britain's benefit; but the courage to combine the original design of two caissons into one of such magnitude had been that of Mr Storey Wilson, and the Paper was a valuable and complete record of what was an astonishing work.

Mr Storey Wilson, in commenting on an earlier Paper on caissons at Leith, had suggested that the names of the engineers who had carried out the work might have been included in that Paper, yet in the present Paper the names of those concerned on his side were missing. Could it be assumed that they had been put in and then eliminated by some authority to whom the Paper had first to be submitted? It was perfectly proper that such an authority should approve a Paper describing work carried out for them, but members of the Institution should be free to mention their fellow-members in their own Proceedings if they wished to do so.

Both Messrs Mouchel and the contractors were to be congratulated on their sound engineering common-sense in allowing the caisson to drift and in concentrating on the final level and not on the final position. Such admirable detachment should be copied by those members of the Institution who, having made a drawing, liked to kneel down and worship it, regardless of the need at times to adjust themselves to circumstance. The method of arriving at the right depth by putting slabs under the sleeper walls should be noted, and could be applied to small caissons by stumps inside.

Pump-houses were essential features of most power stations and usually had to be constructed in difficult ground. At Uskmouth the caisson was a perfect and proper solution to the problem of position, depth, and strata to be penetrated. At Coryton, Messrs Laings had recently constructed a floating monolith and had had to position and sink it on to a prepared bed. The only drawback to that method, assuming that a suitable dock was available for the initial construction, was the provision of a good foundation on which the monolith could settle. At Deptford power station the problem had been ingeniously solved, to suit the harder nature of the soil in Thames gravel and stiff clays, by sinking a number of 18-foot cast-iron circular shafts in compressed air and joining them to the intake tunnels, putting one pump in each well, instead of having a single very large chamber. In the same way, at Shellhaven refinery the pump-house was in a cofferdam 250 feet by 50 feet, constructed inland. In that case, the cause

of the difficulty had been soft alluvial clay with a considerable artesia head of water below it. The artesian head had been relieved by a syste of eleven wells, 80 feet deep, each with a submersible pump.

Pump wells had often been constructed in cofferdams, but in th method, if the ground was water-bearing and fine-grained, difficulti could occur, especially when the pile-clutches failed to hold. Only tho who had seen the clutches of a Larssen pile rolled up like a wood shavin would appreciate what could happen.

In a recent case such a cofferdam had been sunk in fine sand and su ported by concrete frames cast in position as the work proceeded. A nun ber of piles had been found to be out of their clutches, and a rescue par had had to solidify the sand by chemical injections. Fortunately the sa had been just within the limits of possible successful treatment; if t grain-size had been very slightly smaller such treatment would have be impossible. It would be interesting to know what Mr Storey Wilson wou do in such a case. It was a warning to the open-grabbing enthusiasts.

At Braehead pumping station the pump-well was to have been co structed in an 80-foot-square steel-pile cofferdam in sand and gravel, bu owing to a change of design, it had had to be located in a soft saturat glacial silt, penetrating into rock to obtain sufficient depth. The consulti engineers, Sir Alexander Gibb & Partners, had wisely decided to u compressed air for that work, but the time quoted for the delivery of steel caisson at that time had been more than a year, and the decision h been made to drive the steel piles which had already been provided for t other site and construct an air deck between them and so carry out t work in compressed air.

The 80-foot-square cofferdam had been divided into four sections, feet by 20 feet, and steel decks of 24-inch-by-7½-inch joists had been weld to the steel plates, concreted in, and covered with earth kentledge. T steel frames had previously been suspended under the air deck and lower as the work proceeded; about 5 feet of rock had been blasted out at t bottom. The engineers had decided to construct the work in compress air, and had been right to do so, in spite of a local opinion that the co tractors had been making an unnecessary fuss. They had already emph sized the need for compressed air, especially as the glacial silt contain large boulders, and they had been afraid that the piles might be distort by them—a fear which had subsequently proved to be well founded.

The silt, which had been a complete soup in its natural condition, h gradually dried out under the influence of compressed air. The piles h failed to penetrate into the rock, which had been blasted out in compress air for a depth of 5 feet or so below the toes of the piles. It was obvi that if the work had been done in free air no toe-hold on the rock wo have been obtained.

Four gangs had worked 8-hour shifts, with two gangs in each compa ment of 1,600 square feet area. Each gang had consisted of one char

hand sinker and five men, or one man to about 133 square feet of floor area, excavating 3 cubic feet per man-hour.

The number of men excavating in the Uskmouth Caisson had been just sufficient to keep pace with the concrete construction above. The number of men spread over the floor area had given each man 680 square feet to work in. In such work the number of men to be accommodated varied greatly according to the rate of construction but not to the rate of sinking. For instance, in 14-foot-diameter cylindrical caissons it was usual to have five men, which meant one man to every 35 square feet, whilst in a caisson of 1,350 square feet there were 13 men or one to every 104 square feet. If Mr Storey Wilson's skips were of 1-cubic-yard capacity the bulking of the excavated material was about 60 per cent compared with 52 per cent in another caisson in Thames Gravel.

Mr H. Shirley Smith remarked upon the efficient design of the roof plate of the caisson, which was haunched so that only tensile stresses were induced in it by the air pressure. It was signally free from the distressing clutter of stiffeners which encumbered flat steel plates resisting pressure.

A very good model of the caisson steelwork had been made by the contractors and supplied to Mr Shirley Smith's firm. It had been sent to the workshops where it had been of great assistance in showing the fabricators exactly what was wanted. Mr Shirley Smith was wholly in favour of models of that kind being made.

He had been so impressed by the caisson design that he had asked the steel fabricators in Darlington if they could see any way in which it could have been improved, or if there had been any particular difficulties. There had been some, but they were of such minor character that they merely served to emphasize the excellence of the design.

First, a number of plates had been welded between the flanges of channels, in the manner shown in *Fig. 19* (*a*). The workshops would have preferred to run the plates just past the toes of the channels and weld them as shown in *Fig. 19* (*b*). Before welding, the plates could have been brought firmly into contact with the channels by means of small angles, temporarily welded on, and wedges as indicated in the Figure. The reason was that channels as rolled were never exactly the same depth; the arrangement in *Fig. 19* (*a*) necessitated cutting and fitting the plates in each bay and that had entailed extra work and expense.

Another suggested improvement was in connexion with the cone-shaped opening at the bottom of the air shaft. It had been designed as a number of small plates, bent segmentally and welded at the connexions. That had been found difficult to make because the close welds on the small plates tended to pull it out of shape. It would have been cheaper to make a cone out of bent plate in the ordinary manner and to protect the edge by splitting a pipe and welding it on, as shown in *Fig. 20*.

The only other point of any significance which the shops had raised was that of the desirability of avoiding hand chamfering. In some places the

ends of thin narrow plates had had to be welded on to the side of a long thic plate (A in *Fig. 21* (*a*)). The ends of the narrow plates had been chamfere by machine before assembly; but the sides of the long plates had had t be hand chamfered at intervals along their length where the narrow plat butted. That could have been avoided by leaving plate A unchamfere and making a bigger chamfer by machine on plate B as shown in *Fi 21* (*b*). The welding pundits might say that that was quite wrong, but was a quicker way of doing the job, and Mr Shirley Smith thought that would be quite satisfactory.

Figs 19

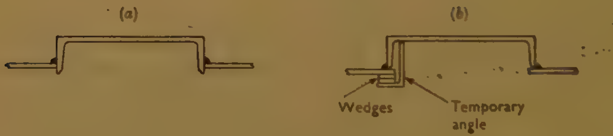


Fig. 20

Figs 21

PLATE B MACHINE CHAMFERED
PLATE A HAND CHAMFERED

PLATE B MACHINE CHAMFERED
NO CHAMFER ON PLATE A

The concrete had been pumped into the working chamber, which wa an excellent innovation. He had been interested to see that the cost o fuel oil for steam raising had been four times that of electricity; that wa worth bearing in mind, but perhaps electricity was cheap at a power station A bonus system had been operated per shift, and he agreed that that wa the best way to get results, particularly today. It was noteworthy tha the final level of the caisson was accurate to within $\frac{1}{8}$ inch. The com pressed-air record had been very good, but he would point out that th

pressure had never risen above 27 lb. per square inch, and it was not difficult to get a good record of freedom from caisson sickness under those conditions.

Mr Storey Wilson had remarked when introducing the Paper that he would like to know how the caisson could have been put down by open dredging. Mr Shirley Smith did not intend to be drawn at such short notice into elaborating a method, although he would have been happy to do so at an earlier date. He had been particularly struck by the little diagram in Figs 14, Plate 2, showing the way in which the caisson had moved out of position. He agreed that the engineers and contractors had been correct in not concerning themselves with the exact position of the caisson, provided that it was kept on the right side of the river! When he had first seen the diagram (bearing in mind all he had been told about compressed air being a panacea for all sinking troubles) he had thought that the contractors had resorted to compressed air when the cutting edge had reached level — 5 feet and thereby halted the slip to the river. Reading more carefully, however, he had found that that was not so; on the contrary, compressed air had been applied at level + 5 feet, which coincided with the commencement of the slip!

Mr B. A. E. Hiley asked what had been the principal reasons for wishing to sink one large caisson instead of doing the work as two units. Had it been the contractual risk of keeping those two units in line, or to save the difficult job of jointing, involving difficult underwater welding and concreting?

The original design for the working-chamber had been in light steelwork, giving vertical walls for the three chambers and a flat roof. The cutting edges had not been inclined in any way, as they were in the alternative scheme (as carried out) which had three chambers with vaulted roofs and flat-pointed shoes. Had the vaulted chambers been introduced because of the desire to incorporate loading and conveyor gear to mechanize and dispose of the excavation, as shown in *Fig. 16*? If not, why had the vaults been used, since the original scheme gave a reasonable method of concrete loading in horizontal layers, as shown on the contract drawings?

From the soil mechanics point of view it had been felt that the flat shoes would help to control the movement downwards. It had even been thought that concrete spreaders under the cutting edges would further arrest downward movement through the upper clay where it occurred and give the opportunity of breaking them up when the more compact ground had been reached. Why had a new type of cutting edge been introduced?

He had followed with considerable interest the records shown on Figs 13, 14, and 15, Plate 2. They showed the drift towards the river, and it seemed to vary considerably. On p. 344 it was stated that "the caisson, when founded, was, on an average, 4 feet 9 inches forward of its original position, or 3 feet 9 inches from its designed position." It was

deduced from that that the datum line for sinking had been — 1·00, an not, as shown on the drift diagram in Figs 14, Plate 2, at 0·00. Further, o the isometric diagram the drift appeared to be shown as 10½ inches. Woul the Authors explain that seeming discrepancy ?

The diagram showed some awkward-looking positions for the stee working-chamber, and the stresses set up in the plated and welded structur must have been severe. Had there been any trouble with the welde seams ?

The contract had provided for implementing the recommendations c the Institution of Civil Engineers for the sinking of caissons. Since thos regulations had been strictly carried out, were there any amendment or additions for which the Authors would ask if they were sinking a secon caisson on the same site ?

Mr Hiley mentioned that he had every reason to believe that th Authors would readily answer the foregoing questions, which were put t them with the idea of stimulating a discussion. He had no wish t detract in any way from the excellence of the contractors' work and skil Indeed, he wished to pay a sincere tribute to them and to the Authors.

Mr N. S. Williams thought that there had been two fundamental facto controlling both the design and the actual execution of the work. Th first was the enormous tidal range in the river, which was of the orde of 42 feet, and had been known during the period of sinking to be as muc as 46 feet. The second was the shear strength of the clays which th structure had had to penetrate.

Boreholes had been put down on the power-station site and the result given by those boreholes had been based on so-called undisturbed sample Later, further boreholes had been put down near the caisson site, and th results had been obtained there by the vane method. At the same tim some information had also been obtained by the so-called undisturbe sample method. There had been a surprising variation in the results. N only was a higher shear strength obtained by the vane method, but th shear strength increased with the depth from the surface downward In Mr Williams's view, the vane method was by far the more satisfactor but very great care had to be exercised in the speed at which the vane wa rotated in clays of the kind in question, because in clays of high fluidit it was possible to develop a very high degree of lubrication and therefo get a comparatively low result.

Considerable care had been taken in the selection of welders, each whom had had to pass a test in proficiency. The welds had first be pickled and then examined by an optical detector. If they had stood up the tests the welder had been taken on, but not otherwise. Througho the whole operation of welding the structure, the foreman welder ha continually to go round examining and testing welds. Mr Willia thought it fair to say that every weld that could be seen had stood u remarkably well to the duty which it had had to perform. It was significa

o appreciate that, at the time the cutting edge had been rolled out, the xtreme end bays had been acting as cantilevers about 55 feet long. That ad been a severe test for the tension welds on the top chord, but they had roved quite satisfactory.

Two or three methods had been adopted for stabilizing the structure luring sinking through various strata, sometimes singly and sometimes in ombination. In general, they had been to control the digging under the utting edge in the places where the strata proved difficult; a greater mount of digging had been done there than in the softer places. In the ase of a very soft place in the south-east corner of the structure, a great uantity of material had been maintained under the roof of the working hamber to support that corner of the structure during sinking. In ddition, it had been arranged that the concrete superstructure work should roceed in such a manner that the greatest weight was applied where he ground was most obstinate. By a combination of all those methods t had been possible to keep the caisson reasonably plumb throughout its ourney downwards.

It had not been possible to construct the intake works on the face of he caisson during the time that the actual structure was being sunk, but t the time of concreting, vertical steel sheet-pile interlocks had been ast into the north face of the caisson. They had been placed in such a osition that, later on, No. 5 Larssen piles 80 feet in length could be conected to them, thus forming a very large watertight cofferdam against ull tide head, in which the intake works were to be constructed.

Mr W. D. Short, as one of Her Majesty's Engineering Inspectors of 'actories, was a member of the Government department responsible for he safety of men engaged on civil engineering work. Almost all civil ngineering work in Great Britain, he observed, and certainly work which nvolved the use of compressed air, came under the Factories Act. That ct required all cases of compressed-air illness to be reported to the District nspector of Factories, so that the Factory Department could assess he risk and endeavour, in consultation with the civil engineers concerned, o decide what should be done to prevent cases of compressed-air illness ccurring.

The regulations, which were mentioned on p. 348, were the result of or 6 years of discussion between civil engineers, the medical profession, nd his own department. The regulations were based on the Institution ules of January 1936, but the Factory Department had found that cases f compressed-air illness occurred too frequently when decompression was arried out at the rates then laid down. The present draft regulations, hich were not yet law, were a compromise between the figures in the nstitution's regulations and the theoretically safe decompression rates.

Theoretically, it was safe for decompression to be carried out on a asis of one atmosphere. The basis of the Factory Department's draft egulations was 18 lb. per square inch and the figure in the old Institution

regulations was 21 lb. per square inch. The results in the case of the Uskmouth caisson showed, he thought, that the new regulations were reasonably good. It was mentioned in the Paper that the draft regulations showed no decompression rates for cases of sickness. The reason for not including those was that it was thought that cases of sickness were best dealt with on medical advice. It had been found by experience, however, that there were two fairly reliable methods.

In the first, which was the quicker, the man was recompressed to the working pressure, kept there for 10 minutes, and then decompressed in accordance with the longest working period in the decompression tables, no matter how long he had been in the working-chamber. The method could be repeated until the pain was relieved. It should be remembered that it was only rarely that all pain was completely relieved by treatment the first time.

An alternative, which was rather slower and could also be used for more serious cases (where the patient suffered from numbness, might have difficulty in walking, or might be unconscious), was as follows. The man was first recompressed to the working pressure which was then maintained for 10 minutes. Then for pressures of 40 down to 30 lb. per square inch decompression was carried out at 1 lb. for 3 minutes, from 30 lb. to 15 lb. at the rate of 1 lb. for 5 minutes, and from 15 lb. to zero at 1 lb. for [illegible] minutes.

Three of the gases mentioned on p. 350 were odourless and dangerous in very small quantities. The fourth, hydrogen sulphide, though normally expected to have a strong distasteful odour, was difficult to detect by sense of smell in some circumstances. Dangerous concentrations of any of those gases might be reached in a very short time and continuous methods of detection were preferable to taking samples at intervals as in the Uskmouth caisson.

Carbon monoxide might come from faulty compressors if they were overheated; cases of that had occurred. Pockets of the other gases might be released during excavation. The position of air inlets to the compressors was important; they should always be outside the compressor house, and preferably fitted with dust filters.

On Form E it was mentioned that the men should go to the rest hut. It had been found by experience that most cases of compressed-air illness occurred within 1 to $1\frac{1}{2}$ hours of leaving the air-lock. Men should be encouraged, therefore, to remain on the site for about $1\frac{1}{2}$ hours when the working pressure had been in excess of 40 lb., and for 1 hour when it had been less than 40 lb.

Mr S. E. Jones observed that he had taken over from Mr R. S. Read, the Agent in the initial period, only when the erection, rolling out, and lowering had been completed and the sinking operations had become more or less straightforward.

The caisson, like all others, had presented him with the usual quota

peculiarities and unexpected behaviour. On such occasions it was most important to be able to take decisions immediately; a caisson, once it started to misbehave, would not wait for a conference to be convened and procedure to be discussed. In that respect the greatest help had been received from the Resident Engineer and his staff in making decisions almost overnight.

The original scheme of using twin caissons had been referred to during the discussion. It would have been seen from the graphs in the Paper that the caisson as constructed, when it had weighed about 3,800 tons, had broken through the hardcore shell and had started a downward dive. Allowing a load spread of 45 degrees through the 6-foot-6-inch-thick hard-core, that worked out at a failing load on the virgin ground of about 6 cwt per square foot. It seemed to Mr Jones that under any conditions it would have been very difficult to construct a reinforced-concrete shoe on ground which could take only 6 cwt per square foot.

The Authors had given reasons for the caisson drifting outwards towards the river. If those reasons were correct, as Mr Jones believed them to be, then he could see no possibility of preventing twin caissons (had they been used) from behaving in accordance with some sort of inverse law of gravity and repelling one another to a limit which it was impossible to guess.

On the design of the steel shoe, he would like to be wise after the event and suggest that perhaps it would have been better if the steel skin plate on the outside had been higher by a foot or two, or possibly more. On one occasion in the early stages, after the caisson had made its first dive downwards, there had been a very grave risk of the water flowing over the top of the skin plating before the reinforced-concrete walls could be started.

The problem of ensuring that the caisson was finally level and plumb had been relieved considerably by its size. It had obeyed the well-known law that a caisson could be rocked about any of its main dimensions, always provided that the depth of penetration was less than either dimension of the plane about which it was desired to rock it. In the case in question, the maximum penetration had been 64 feet and the main dimensions were 164 feet by 110 feet, so that right to the last it had been possible to rock it into level. Had there been twin caissons the penetration would probably have been more than one of the main dimensions so that it would have been impossible to rock them after they had reached a certain depth.

He suggested that the value of 10·3 cwt per square foot given by the Authors for the skin friction savoured more of "adhesion" than "friction."

Turning to safety and welfare, Mr Jones pointed out first that Form G (shown in the Appendix) was a report form drawn up to record individual cases of caisson sickness. It had been found on the site that it was a useful addition to put in the number of the lock through which the man had passed when leaving his last shift, and also the identity of any other men who had passed through that lock with him at the same time. That provided a

check on the susceptibility of the man who suffered from caisson disease, and also a check on the accuracy with which the lock-keepers had let the men out of the compressed air. If it were found that two or three men had caisson sickness coming out of the same lock, it could be said that the lock-keeper was probably responsible.

Secondly, under the regulations there were two alternatives for taking men into compressed air. Either a man was allowed to let himself into the air (and a group of men were allowed to choose a leader who let the group into the air) or the increase of pressure was under the control of an outside lock-keeper. Mr Jones maintained that the safer way was for people to go into the air under their own control. That was the method used at Uskmouth, and Mr Jones himself had been thankful for the chance of controlling his own entry into the air when troubled with severe sinus pain.

Mr C. D. C. Braine observed that, for some time past, he had been actively engaged on the design of an unusually deep and large pumping station, the caisson for which would be somewhat shorter than that at Uskmouth; its width was about half, but its depth from cutting edge to ground level would be a good deal greater. He had long ago come to the conclusion that the only practicable method of constructing such a deep pumping station would be to adopt a heavy cellular form of structure, such as that used at Grangemouth,[2] and to sink the whole of it as a caisson more or less in the way described by the Authors.

Having constantly urged that the design should be viewed from the contractors' angle all the time, he felt that the present Paper was opportune, since in it had been set out the methods adopted by a contractor for sinking a very large caisson of the type about which he had been thinking. The decision to sink one large caisson instead of two seemed to be the right one, but it would be interesting to know if, after the event, the contractors had ever had second thoughts about it. From what Mr S. E. Jones had said earlier in the discussion it seemed probable that they had not, but there might be other views.

Some of the construction procedure seemed rather surprising, particularly having regard to the fact that the contractors had put in an alternative tender, which had surely given them a good deal of scope. It might be, however, that some of the steps taken had been dictated by site conditions of which Mr Braine had no knowledge. For instance, looking at *Fig. 1*, it might be assumed that the site just south of the pumping station was permanently available, in which case he would like to ask whether it would not have been cheaper, simpler, and safer to have constructed and sunk the caisson there on the river bank above high-water level. The cutting edge could then have been constructed in reinforced concrete, which was common practice, thus saving the need for the steel

[2] C. W. Pike and B. F. Saurin, "Buoyant Foundations in Soft Clay for Oil-Refinery Structures at Grangemouth." Proc. Instn Civ. Engrs, Part III, vol. 1, p. 301 (Dec 1952).

cutting edge described in the Paper, with its accompanying high cost of erection and subsequent launching. He had heard gossip before the meeting to the effect that the actual steel in the cutting edge had been less than that which would have been used had it been a reinforced-concrete caisson. He did not know whether that was a fact, but if so it was a very interesting one. Apart from the steel itself, the cost of the falsework required had been quite considerable.

If the caisson could have been re-located as suggested, on the top of the bank, it would be interesting to know whether it would or would not have been sunk in free air. His own impression was that free air might well have been used. There had obviously been many discussions in the Authors' office about the advisability of using air, and it would be very interesting to know some of the opinions expressed. It would also be interesting to know if the Authors, had they been going to sink a similar caisson in the East, would have chosen free air or compressed air. His experience had been that the human element in the East was subject to more vagaries than in Great Britain.

Some years ago, one of the leading bridge contractors in Britain had told him that, so far as bridge caissons were concerned, his firm almost invariably elected to sink them using compressed air, because, although air often cost more, it was much safer and the caissons could be kept under control very much better. With a caisson of the Uskmouth dimensions, however, one would not have expected steering to create serious difficulty, and from what had been said it would appear that it had not done so. On the other hand, unit costs increased considerably as working pressures increased; perhaps the Authors could give some indication of that percentage increase in unit costs at the Uskmouth works. Judging by past experience, actual costs seemed to increase faster than was expected.

The Authors had stated that a depression had formed around the caisson as it sank, and they had attributed that to shear effects as the caisson went down. It might be asked, however, how much of that sinkage could be attributed to ground outside the caisson being drawn into it. Mr Braine imagined that the width of the depression increased as the caisson sank, and he would like to know approximately what angle the edge of the depression had made with the cutting edge of the caisson, and what had been its depth. Presumably, just outside the depression the ground was stable, but within it it was unstable. Inspection of Figs 2, Plate 1, showed that the circulating-water and cable ducts were supported on long vertical piles which passed through the apparently unstable ground, and they would be too slender to stand up to any ground movement of that nature, so that it would be interesting to know what happened. On the shore side there must have been a considerable area of instability, because the caisson moved bodily 4 feet 9 inches towards the river. In the circumstances, it was almost surprising that the ducts had not been subsequently cantilevered out from the caisson and the piles omitted.

He noted that the contractors had used electrically-driven compressors with steam as a standby, but, although they had had to contend with four electrical failures, no harm had resulted. The Chairman would recall a case where the contractors had applied to use electricity instead of steam, steam having been specified on account of its reliability. The concession had been granted, and a few days later it had happened that a cat chasing a mouse in the local power station, or something of that kind, had caused a blackout which covered much of south-east England. Before the standby plant could be brought into operation there had been a very serious "run" in the tunnel, and he believed that a tram had fallen into the considerable subsidence which had resulted. The question arose, therefore, of whether or not electricity was sufficiently reliable to be used for work of the kind in question.

Mr R. W. A. Fane commented upon the astonishing amount of sound planning and forethought which had ensured that the job had gone through without, apparently, any serious hitch. That in itself was a matter for which extreme credit was due to those responsible. He had had the privilege of seeing something of the work in the early stages, and he would like to ask one or two questions, based on inside knowledge, in the hope of drawing from the Authors a little more information about the "whys and wherefores" of some of the earlier decisions which had been made, and which in every case appeared to be justified by the results.

One of the main difficulties on the site had been the nature of the soft silt. The film shown by the Authors when introducing the Paper had illustrated that better than any verbal description possibly could. In order to get a sound foundation when the caisson was first lowered, the site had been excavated in front of the bund and a layer of reject bricks from the local brickworks had been put down with a layer of aggregate on the top. Would the Authors give their views on the success of that scheme from the point of view of giving them initial control of the caisson before they had been prepared to "let her go"?

In the light of Figs 14, Plate 2, the recommendation to sink one big caisson, instead of two smaller ones with a 3-foot space between, had been completely justified, on the lines of Mr Jones's remarks about what might have happened to the two caissons—or rather to the gap between them.

The Authors had displayed a model of the caisson as constructed, but Mr Fane hoped that they would say something about the earlier model referred to by Mr Shirley Smith. Mr Fane well remembered the engineers struggling with a three-dimension tangle of steelwork and trying to sort it out for themselves with pieces of cardboard. That had developed into the earlier paper-and-cardboard model, which must have contributed very much to the solution of many of the problems which Mr Sully and his colleagues had had to deal with when designing the steelwork.

No mention had so far been made of the earlier caisson at the Newport East power station, about a mile further up the stream. He believed

that that power-station caisson, which had been put down about 30 years previously, had moved outwards to the river a distance of about 4 feet. Could the Authors say a little more about that caisson ?

Mr Storey Wilson, in reply, observed that the Chairman might have appeared to give him personally some credit for the job, but it had been, in fact, very much a joint effort on the part of all the people in Messrs Holloway's office in scheming out the whole job, and credit could not be given to one or two people only.

Mr Harding had asked why no acknowledgements were included in the Paper. In the draft of the Paper acknowledgements had been made to many people, but higher authority had called for their deletion. He had accordingly deleted them, and he would like to express his apologies to all those whose names had not been included in the acknowledgements.

Mr Shirley Smith had made some points about the caisson having moved forward 4 feet 9 inches. They had not minded that. Mr Storey Wilson had asked the consulting engineers " Would you like to have a caisson sunk to the correct level and plumb, or would you like one sunk exactly in the position shown on the drawings ? " The reply had been : " You can have it where you like, provided you do not go into the middle of the river, so long as it is plumb and at the correct level." They set out, therefore, to get it plumb and at the correct level, and that was what they did. *Fig. 22* helped to explain why it had moved forward.

The ground was very soft and the whole of the working chamber had filled with material. The weight of the caisson had imposed a pressure on the material which had flowed out under the cutting edge and had been washed away on the river side, but as it had flowed out in the direction shown by the arrow, it had set up a pressure between the caisson and the sheet-piling which had caused the caisson to move forward. It would be convenient, at the same time, to answer Mr Hiley's question about the two concrete caissons. If there had been two concrete caissons, as shown in *Figs 23*, the material would have flowed out in the direction shown by the arrows, and a pressure would have been set up between the caissons, causing them to move apart and create a wedging action ; he would not like to say how far apart they might have gone. Mr Hiley had asked what were the principal reasons for wishing to sink one large caisson instead of two units, and whether it had been the contractual risk of keeping the two units in line. That risk, in Mr Storey Wilson's opinion, would have been such that he would not have taken it, but would have allowed the job to go to the lowest tenderer !

Mr Hiley had referred to the fact that the alternative scheme, as carried out, showed three chambers with vaulted roofs and flat-pointed shoes, and had asked whether the introduction of the vaulted chambers had been due to the desire to incorporate loading and conveyor gear to mechanize and dispose of the excavation. That was not the case. Mr Hiley had also asked why, since the original scheme gave a reasonable

Fig. 22

Figs 23

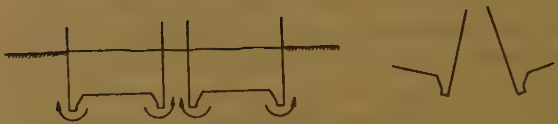


Fig. 24

Fig. 25

Fig. 26

method of concrete loading in layers, the vaults had been used. The answer was that by having the vaults and placing very little concrete in the position marked by the arrows in *Fig. 24*, a large settlement had been obtained and it had been possible to move down a long way, whereas if one had had a caisson of the kind shown in *Fig. 25* and the concrete had been put in as shown, a great deal more concrete would have been required and the weight would have been much greater for the same depth of settlement. That was one reason. The other reason—the really important one—was that, in the early stages with the vaulted construction the plates had been in pure tension with compressed air in the working chamber (*Fig. 26*).

He thanked Mr Williams for the very kind remarks made about the contractors. Mr Braine had also raised the question of the use of two caissons compared with one. In Mr Storey Wilson's view there was no doubt that from a practical point of view one caisson was the correct solution. A reinforced-concrete caisson of only half the size of that actually employed would have weighed 4,000–5,000 tons before the working-chamber was built, and the ground would have been unable to take such a load. It had been necessary to have a very light caisson in the early stages and load it up slowly as the caisson sank, which had been the determining factor in deciding to have a steel caisson.

Mr Sully, in reply, agreed with Mr Shirley Smith that the detail shown in *Fig. 19* (*b*) was preferable; that was borne out in the site welding and would have had the further simplification of avoiding edge planing of the plates.

The flared cone-shaped opening at the bottom of the air shaft was admittedly very difficult to construct by welding. In order to deflect the bucket rope round an easier curve than that shown in *Fig. 16* or *Fig. 20*, the Authors had decided on future work to provide a cast-iron segmental ring, which should be more satisfactory.

The maximum air pressure recorded had been 33 lb. per square inch.

Mr Shirley Smith had pointed out that the slip towards the river had occurred mainly between cutting-edge levels +5·0 and −5·0, whilst the compressed air had been put on at +5·0. During that initial period the working chamber filled up with the very soft clay which squeezed up the working shafts, the whole caisson at that stage behaving as if it were floating on a mass of jelly. The great difficulty had been to remove the material through the air-locks at a fast enough rate to lower the caisson to firmer strata while the outward drift was taking place.

In answer to Mr Hiley, the original design for the twin caisson steelwork consisted of open lattice girders with plated bottom booms to form a base for placing concrete, without any vertical projection below the plate. Those girders had been designed to give structural rigidity on the "Melan" principle since the concrete work in the haunches and roof was placed inside formwork, additional bar reinforcement being provided where necessary. The two caisson structures were to have been built on a prepared bed on the foreshore inside a temporary cofferdam. The Authors considered that the low shear value of the top material rendered that a very hazardous proceeding and preferred to have one large caisson shoe in the form of a completely plated steel shell which would float and give the maximum rigidity for least weight.

Mr Storey Wilson had explained the reason for the vault construction, the main object being to limit, so far as possible, the dead weight of wet concrete of little strength in the early stages.

The use of sloping haunches to the inner faces of the caisson shoe, as opposed to the more nearly vertical faces proposed in the original scheme,

was not new but was in conformity with British and American practice, it being much easier to undercut during sinking and avoid a possibly disastrous collapse due to failure of the subsoil, the sloping face giving increased resistance as the cutting edge penetrated the ground, and bringing the structure to rest more quickly.

The smaller diagram of Figs 14, Plate 2, showed the drift forward from the position as set originally, the true position as designed being 1 foot forward from the zero on that diagram. The drift was in a northerly direction, and the 10½-inch dimension given was the final movement to the west of the initial position as set.

The isometric views of the plane of the cutting edge in Figs 14, Plate 2, were shown to an extremely exaggerated scale. The total amount of " wind " in the plane of the cutting edge, as measured across from points 1 and 12, was never more than about 2 inches. There was no sign of excessive stress in any of the welding.

The remarks made by Mr Williams with regard to the borings were very sound. As usually happened, the preliminary borings taken for the pump-house were rather scanty in number and inferences had to be drawn from the other test bores taken for the site, but not in the immediate vicinity of the work.

Referring to Mr Braine's remarks, the position of the pump-house had been determined by the lay-out of the power station as a whole and the position indicated by Mr Braine had not been available. If it had been available, the sinking of the caisson would have been much easier. With regard to the amount of steel in the caisson (including all plating and details) in comparison with the amount of concrete in the caisson sho (excluding all working-chamber concrete), the ratio was 1 ton of steel to 12·5 cubic yards of concrete.

If the pump-house had been constructed behind the bund, as suggeste by Mr Braine, the Authors would still have sunk under compressed air Probably it could be sunk as an open well, but an air deck would hav been required to bottom-up. With that method, a lot of the interio details would have had to be delayed and completed after founding th structure and, during the period of sinking, by grabbing, it would be fa less under control as to level and verticality, which were of vital import-ance.

The Author's firm had sunk more than fifty caissons under compresse air quite successfully in the Middle East for various foundations, usin local labour.

From a careful comparison of the excavation quantities it could b stated that the amount of " run-in " had been practically nil. The angl of depression at the face of the caisson had been approximately 1 in 8 and the depth of the water at the face at low tide about 2 feet.

The question of cantilevering the ducts had been considered bu abandoned, because supporting brackets of considerable size would hav

been required below ; it had been feared that they would cause difficulty in the sinking operations. If the Authors were repeating the work, they would recommend increasing the size of the caisson, if necessary, to include the ducts and also the entrance gates.

There was one point that had not been commented upon ; that was the high figure of 10·3 cwt per square foot for skin friction. Previous figures had been given for the Baghdad Road Bridge Caissons,[3] where the skin friction did not exceed 4 cwt per square foot, and for the Kafr el Zayat Bridge [4] in Egypt, where it had been more than 6 cwt per square foot. In the Author's opinion, the high value obtaining at Uskmouth was attributable to the time taken in sinking the caisson—about 9 months—sufficient for considerable ground consolidation to take place, as against the 4 or 5 weeks taken to sink the other caissons in the Tigris and the Nile.

Correspondence on this Paper is closed. No contributions may now be accepted.

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[3] A. E. Reid and F. W. Sully, " The Construction of the King Feisal Bridge and the King Ghazi Bridge over the River Tigris at Baghdad." Works Construction Paper No. 4, Instn Civ. Engrs, 1946.

[4] K. E. Hyatt and G. W. Morley, " The Construction of Kafr el Zayat Railway Bridge." Proc. Instn Civ. Engrs, Part III, vol. 1, p. 101 (Apr. 1952).

STRUCTURAL AND BUILDING ENGINEERING DIVISION MEETING

6 May, 1952

Professor A. J. Sutton Pippard, M.B.E., D.Sc., M.I.C.E., Chairman of the Division, in the Chair.

The following Paper was presented for discussion and, on the motion of the Chairman, the thanks of the Division were accorded to the Author.

Structural Paper No. 32

"The Historical Development of Structural Theory"

by

Stanley Baines Hamilton, Ph.D., M.Sc., B.Sc.(Eng.), M.I.C.E.

SYNOPSIS

Building is one of the most ancient of the practical arts, and was conducted with great skill and marked economy of material in some of the notable buildings of the late middle ages; but structural theory in the modern sense was not possible until the laws of static equilibrium had been clearly formulated. These laws were applied to isolated structural problems in the first half of the eighteenth century, but it was the work of Euler and Coulomb in the latter half of that century that formed the foundations of modern statical theory. Towards the end of the eighteenth century, a few French engineers began to use the testing machine in their study of structural materials. Navier, in the eighteen-twenties, practically completed the statical theory of structures, and laid the foundations of the mathematical theory of elasticity.

The use of iron as a material of construction led British engineers, early in the nineteenth century, first to pay attention to, and then to extend, the theory which had hitherto been studied almost exclusively in France. In the first published works on the forces in framed structures, which appeared about 1850, each node point was treated separately by resolution of forces. In the eighteen-sixties, a few technical writers studied the forces in a framework as a whole by means of reciprocal diagrams; this study led to the development during the following decades of a complete system of graphic statics. Meanwhile, strain-energy methods, hitherto regarded as too academic for practical use, were applied to the determination of the moments at the joints and the forces in the members of stiff-jointed and redundant structures. In all but the simplest cases, however, computation by these methods was extremely laborious. Recent trends show favour for methods of successive approximations, for the use of models, and the saving of time and labour by mechanical computation. Current developments involve consideration of the stability of frames when some of their members are stressed beyond the elastic limit, and measurement of strain in full-scale models and actual structures.

Introduction

Textbooks on the theory of structures quite rightly concentrate on the exposition of the best available methods of solving structural problems,

and in so doing pay little or no attention to the process by which these methods were developed, or to the personalities of those from whose work the finished method was derived. The process of development has, however, an interest of its own; and those engaged in extending the field of application of mathematics and mechanics to engineering problems could gain a deeper insight into the meaning and limitations of the previous work they take for granted, and also humanize their sometimes arid labours by considering how, in what stages, and by whom the body of knowledge which they have inherited was compiled. This point of view has been more widely appreciated by American than by British authors and teachers. Among the more accessible sources may be mentioned a valuable Paper, fully documented, by H. M. Westergaard [1] of Harvard (formerly of the University of Illinois); the chapter entitled "General Discussion and Historical Review" in a treatise by J. I. Parcel and G. A. Maney [2]; and a book by L. E. Grinter [3] of the Illinois Institute of Technology, which begins with an outline of the history of construction in steel and later reviews the classical methods of analysing indeterminate structures. British sources include the classic works on elasticity by A. E. H. Love (whose book [4] has an historical introduction) and by I. Todhunter and K. Pearson,[5] and a Paper by A. A. Fordham [6] of Swansea University. There are differences, however, in the scope and attitude of these authors, and, having studied their works, the Author feels there is still room for the present Paper.

Structural practice is one of the most ancient of the technical arts, and it would not be inappropriate to begin this Paper by attempting to reconstruct the empirical rules by which, for instance, mediaeval master-masons balanced great loads with remarkable economy of material, for it was against the background of their experience that modern theory began to take shape in the seventeenth and eighteenth centuries. A full treatment of the history of theory should also trace at every stage its links with current methods of design and construction, though the full story of their inter-relations would involve further research. Within the limits of a single Paper, however, only an outline of the origin and main trend of development of ideas and methods employed in modern analysis and design is possible. Ancient history, and all but the most significant links with other lines of thought and activity must, however regretfully, be omitted.

The Foundations of Structural Theory

Structural theory, in the sense in which the term is used by engineers today, could only begin to take shape when the laws of static equilibrium had been clearly formulated. Though foreshadowed in the "*Mechanica*" of Aristotle,[7] the notebooks of Leonardo da Vinci (1452–1519),[8] and the

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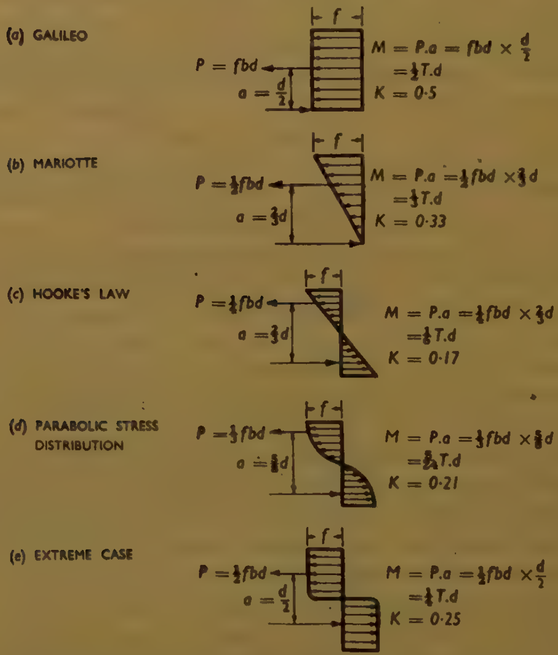
[1] The references are given on p. 399.

writings of Simon Stevin (1548–1620),[9] the parallelogram of forces was only given its general form as a corollary to the Principal of Virtual Velocities by Galileo Galilei in 1638[10]; and the Principle of Moments was first stated by Pierre Varignon in 1725.[11]

Galilei also initiated the study of the strength of materials (one of his

Figs 1



M denotes Moment of Resistance under extreme fibre stress f in a beam of rectangular section

P denotes resultant longitudinal force in tension or compression

T „ "Absolute Resistance" or ultimate tensile strength of the section of area $b \times d$

K denotes the ratio of M to $T.d$

Moment of Resistance of Rectangular Sections in accordance with Various Hypotheses.

"two new sciences"—the other was dynamics) when he proved that th "resistance" (moment of resistance) of a beam was proportional to it breadth, to the square of its depth, and inversely proportional to its spar He thought, however, that all the fibres, except at the compression edg which formed the fulcrum, were in tension and equally stressed (see *Figs 1* Edmé Mariotte also treated the forces in all the fibres as tensile but a ranging in intensity from zero at the fulcrum to the maximum at the conve

face.[12] Robert Hooke (1635–1702) enunciated his famous law—"*Ut tensio sic vis*"—in a lecture in 1678,[13] but its importance and generality were unrecognized until these were pointed out by Dr Thomas Young at the beginning of the nineteenth century, and adopted as the basis of the mathematical theory of elasticity by C. L. M. Navier in 1821.

Jakob Bernoulli (1654–1705) derived the general equation for the curvature of a bent beam, introducing the assumption that a plane section remained plane after bending.[14] He was probably unaware of Hooke's work. He found that a twisted cord of gut stretched proportionately *less* as equal increments of load were applied, and argued that the same must be true for material under compression, for were it otherwise, if a certain load compressed a material to half its original length, double that load would make it disappear altogether, which he said, was absurd!

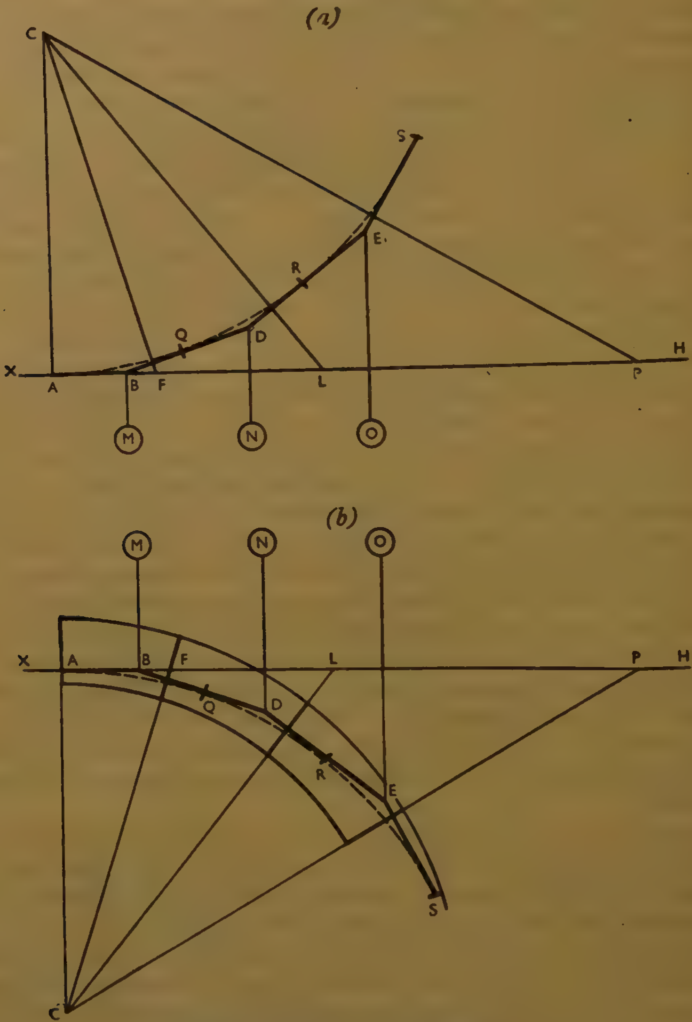
An interesting anticipation of the use of force and link polygons as reciprocal diagrams appears in a short treatise,[15] published in 1695, by Philippe de La Hire (1640–1718). It will be seen in *Figs 2* that CAF, CFL, and CLP are respectively the triangles of forces for the points B, D, and E on a loaded chain or cord. Turned upside down, La Hire's stretched cord became the line of thrust normal to the meeting faces of the voussoirs of an arch. La Hire's Smooth Voussoir Theory of the stability of the arch was accepted as sound until nearly the end of the eighteenth century.

In 1691, Pierre Bullet (1639–1716) suggested that the stability of a mass of earth was much the same as that of a heap of round shot, and on this basis computed, though somewhat inaccurately, the force exerted by earth on the back of a retaining wall.[16] This idea was elaborated by Pierre Couplet (d. 1744) in a memoir which he presented in 1726 to the Académie des Sciences.[17] In practice, however, engineers continued to design retaining walls to the table of empirical dimensions drawn up by Maréchal de Vauban.

René Ferchault de Réaumur (1683–1757), commissioned by the Académie to compile a description of Arts and Crafts, produced in 1722 a volume in which he described a rough bending test for tenacity and a qualitative indentation test for hardness.[18] The first recorded accurate measurements of the strength of materials were made, however, by Petrus van Musschenbroek (1692–1761), whose family were celebrated instrument-makers in Leiden, notably to s'Gravesand and Boerhaave, who were then active in introducing experimental science into their University teaching.[19] Musschenbroek constructed a small lever-loaded testing machine, with grips specially devised to avoid eccentric loading. On this he made tensile tests on small specimens, up to about $\frac{1}{4}$ inch square in section, of various woods, glass, and other materials, and on metals in the form of wire. He also tested small beams, carefully noting the effect of end fixity, and columns with various ratios of length to diameter.

The need for specific training in technical matters was first realized by the military authorities in France, and training colleges for officers were

Figs 2



La Hire's Link Polygon applied to :—
(a) A Hanging Chain
(b) An Arch with Smooth Voussoirs

founded, such as that at La Fère to which Bernard de Belidor (1693–1761 was appointed Professor of Mathematics. So impressed was Belidor b the lack of suitable textbooks that he proceeded to compile short treatise on architecture (1720) and mathematics (1725), and later wrote his classi works on military and hydraulic engineering.[20] The two latter work covered the whole theory and practice of engineering at that period an

remained standard works until well into the nineteenth century. Belidor made simple common-sense applications of mechanics, with the minimum use of mathematics.

The Ecole des Ponts et Chaussées was founded in 1747 and, from 1750 until the Revolution, came under the directorship of Jean Rodolphe Perronet (1708–1794), who also held the post of Premier Ingénieur du Corps des Ingénieurs des Ponts et Chaussées. The Ecole served the double purpose of a training establishment and a professional institution within the organization of the Corps. From that time onwards, in France, entrants to the professions of military engineering, civil engineering, and architecture, between which no clear line of demarcation had hitherto been drawn, were separately recruited and trained.

The Royal Military Academy, the first technical school in Britain, was founded at Woolwich in 1741, with John Muller (1699–1784) as headmaster. Muller wrote a textbook on mathematics, and in 1755 a treatise on fortification [21] that was largely based on Belidor.

The Work of Euler, Coulomb, and Young

Leonhard Euler (1707–1783) extended Jakob Bernoulli's work on the elastic curve, and in 1757 submitted his celebrated Paper on the strength of columns to the Academy of Berlin.[22] Euler found mathematically that a long thin straight column of homogeneous material, when axially loaded, would suffer no deflexion whatever under a gradually increasing load until that load reached a critical value, P_E, at which the column would become elastically unstable. P_E can be expressed thus:—

$$P_E = \frac{S \, . \, \pi^2}{L^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where S denotes the *moment de raideur* (moment of stiffness) and L the length of the column. In a paper presented in 1778 to the Academy of St Petersburg, Euler showed that S had the dimensions of a "moment of inertia" since it could be expressed as the product of a force and the square of a length. The force, in modern terms, is EA, where E denotes "Young's Modulus" and A the area of cross-section. The length was denoted by g, the "radius of gyration" of the section. In modern works, however, it is Ag^2, the second moment of the area, which is less happily referred to as the moment of inertia (I) of the section.

Euler, like Galilei and Bernoulli, placed the axis of the column section at the extreme edge of the section. It was Dr Thomas Young (1773–1829) who showed that the neutral axis of a beam section passed through the centroid, and that, if a rectangular column was loaded with an eccentricity e, measured from the centroid in the direction of the depth d, the "neutral axis" would be at a distance of $\frac{d^2}{12e}$ from the centroid, generally right outside the boundary of the column.[23] Young also cleared up the paradox

whereby deflexion under an increasing endwise load remained zero until the load attained the critical value at which the deflexion became indeterminate. This he did by proving that, if the central axis of the column was originally slightly bent in the form of a sine curve with maximum divergence e at mid-length, the deflexion Δ under any load P would be

$$\Delta = e \cdot \frac{P_E}{P_E - P} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2a)$$

Slightly rearranged, this equation becomes

$$\Delta\left(1 - \frac{P}{P_E}\right) = e \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2b)$$

for which Δ and P plot as a rectangular hyperbola. Euler's column represents the extreme case in which $e = 0$ and the hyperbola coincides with its asymptotes (see *Fig. 3*). Young's formula was rediscovered—presumably independently—by Professor John Perry a century later and has entered twentieth-century technical literature as the " Perry-Robertson " formula. (See Appendix.)

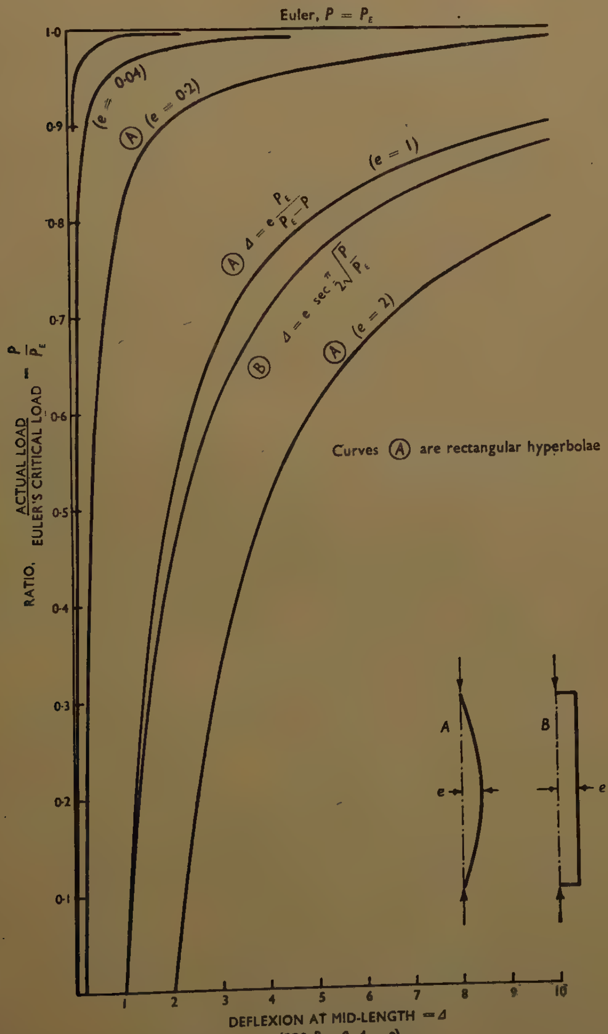
Of all eighteenth-century engineers, modern structural theory is most deeply indebted, however, to Charles Augustin Coulomb (1736–1806). The Author has dealt specifically with Coulomb's contributions elsewhere so that reference here need only be brief.[24] In 1773, Coulomb presented to the Académie des Sciences his well-known brief essay [25] in which, after a clear statement of the basic laws of equilibrium, he applied Euler's mathematical device of differentiating for a maximum, and thus introduced the following important innovations into structural theory : the frictional-shear theory of failure in compression ; the relationship between the applied load and the bending moment, moment of resistance and shearing force in beams (as still taught in modern text-books) ; the wedge theory of earth pressure (as still used) ; and the treatment of the arch, not as an assembly of smooth voussoirs liable to slide on one another, but as a body liable to crack and so form virtual hinges about which the parts can rotate. Coulomb applied his beam theory only to rectangular sections, for which he showed the neutral axis correctly at mid-depth. The theory of beams was later generalized by Navier.

Coulomb left his earth-pressure equations in algebraic form. Their transformation into trigonometric symbols was made by Reinhard Woltmann.[26] In the trigonometric form they were adopted by K. Mayniel,[27] and by G. C. F. M. Riche de Prony (1755–1839) in his works on earth pressure, and so used in the Ecole des Ponts et Chaussées of which Prony was head for many years.[28] Français extended the theory to include the analysis of a clay bank sloping upwards from the head of the wall.[29] J. V. Poncelet (1788–1867) included in his main formula friction at the back of the wall, which Coulomb had applied as a correction.[30] He also developed a graphical construction to find the length of the line of which

the square, multiplied by half the density of the earth, gave the total earth pressure.

Alexandre Collin (1808–1890) studied in the field the curved surface, approximately cycloidal, along which slip actually takes place in a bank of clay. Rankine analysed the stress distribution within the earth in terms of conjugate pressures, but with the incorrect assumption that the resultant

Fig. 3



Deflexion of Columns with Slight Defect from Euler's assumption of Central Loading

thrust, which in the mass of earth is always parallel to the slope of the bank behind the wall, is so transmitted to the back of the solid wall. Nothing much of importance was added to the theory of earth pressure until the modern study of soil mechanics was taken up after 1910.[31]

Coulomb's essay received less immediate attention than it deserved, probably because Perronet, the head of the profession, was an ageing man and disinclined to revise his methods at the behest of a young unknown military engineer. Coulomb's ideas did, however, influence some of the younger men such as E. M. Gauthey, de Prony, and Navier, and through their writings came to permeate the engineering literature of early nineteenth-century France. Coulomb's work was also carefully studied by Dr Thomas Young, whose "Natural Philosophy" (1807) and articles in the Supplement to the 5th Edition of the Encyclopaedia Britannica, though difficult reading, were not without influence on Robison, Barlow, and other English authors of technical works.

Coulomb did not pursue further the subjects treated in his essay, but conducted investigations into the means of carrying out engineering works under water by using devices such as the diving bell; into the frictional losses in simple machines; and into the torsion of wires. Even more famous were his fundamental studies in magnetism and electricity. Coulomb applied his torsion theory to the calibration of the torsion balance which he devised to measure very small electric and magnetic attractions. The application of Coulomb's theory of torsion to shafting was long delayed, but once accepted has, except where torsion and bending act together, never been superseded. His discovery that a twist of large amplitude given to a wire could produce a permanent set without destroying the elastic property of the material led Coulomb to suppose that large distortions caused the formation of slip planes. Another century was to pass before Sorby, examining sections of overstressed metals under a high-power microscope, was able to see the slip planes and thus to verify the correctness of Coulomb's supposition.

Extension of Theory through the Use of the Testing Machine

So long as the materials of construction were limited to masonry, wood, and earth, used according to traditional methods, no elaborate theory was needed, and even Coulomb's simple analyses were avoided as being too mathematical. Belidor had written all that was considered necessary. J. G. Soufflot (1713–1780), however, broke with tradition when he embodied a coarse mesh of wrought-iron bars in the masonry of the monumental classical temple which he built to replace the ancient church of St Geneviève in Paris and which later became the Panthéon Français. Soufflot claimed that the presence of this metal "armature" justified the erection of a lofty dome on four small clusters of columns without the flanking bastions traditional in such construction.[32]

E. M. Gauthey (1732–1807) favoured Soufflot's principle but realized that to carry conviction his argument must be backed by knowledge of the carrying capacity of the stone columns. Musschenbroek's tests had not included samples of stone, and Gauthey therefore constructed a substantial wood-framed lever-loaded testing machine powerful enough to crush specimens of stone of sizes up to 8 inches long and 2 inches square in section. An improved machine with an iron frame, steel knife-edge bearings, and a screw-jack to apply the load and maintain the weigh-beam floating level was built by Jean Rondelet (1734–1829). A similar but more powerful machine was installed by Perronet in the Ecole des Ponts et Chaussées.[33] It was not until 1818, however, that an account was published by George Rennie of a machine, built by him, which was capable of crushing specimens of cast iron—which had by then assumed considerable importance as a material of construction.[34]

Bridges with cast-iron arches, and "fire-proof" mill-buildings with columns and beams of cast iron and stone-flagged floors carried on brick arches, were erected in small numbers before the Napoleonic wars, and when the demand for cannon and shot diminished at the conclusion of hostilities, the iron founders, naturally anxious to find alternative uses for their products, were ready to meet the demand for structural ironwork. It was the custom to apply a proof-load to floor girders before they left the foundry, but the testing of columns and arch ribs presented difficulties. Alarming failures were all too frequent, and it became clear to progressive engineers that a sound basis of design in the new material was urgently required.[35] William (later Sir William) Fairbairn (1789–1874) carried out many tests in his works in Manchester and, finding Eaton Hodgkinson (1789–1861) already working on similar lines, in 1827 he placed the facilities of his works at Hodgkinson's disposal. Hodgkinson, as the result of a long and varied series of tests, discovered the most economical form for the cast-iron beam and stressed the importance of knowing the true position of the neutral axis. His ideal beam had a small top flange, a low neutral axis, and a broad bottom flange to take advantage of the high compressive strength of the material while keeping tensile stress low. The breaking load on a beam could be calculated from the formula :

$$W = \frac{cad}{L} \qquad \ldots\ldots\ldots\ldots \qquad (3)$$

where W denotes the breaking load in tons applied at mid-span
a ,, the area of the tension flange in square inches
d ,, the depth of the beam in inches
L ,, the span of the beam in inches
c ,, a constant = 26 for the form of maximum economy.

Modern investigators have found that Hodgkinson's constant, though appropriate for small beams carefully cast and slowly cooled, gives too high

a computed strength for large beams cast under commercial foundry conditions where cooling may be uneven and residual stresses high.[36]

Hodgkinson continued his investigations to include the strength of columns.[37] He tried to summarize his test results in an equation for the critical load in the form:

$$P = \frac{cd^n}{L^m} \quad \dots\dots\dots \quad (4)$$

According to Euler, $n = 4$ and $m = 2$, but to fit the experimental results these indices were too high, and Hodgkinson substituted other indices, such as $n = 3{\cdot}7$ and $m = 1{\cdot}7$. Further investigation, however, particularly with short stout columns had by 1840 convinced him that no single-term formula would fit all cases, but that the actual strength of a column approximated to the harmonic mean between its resistance to crushing and to buckling. The resulting formula, with the coefficient (α) adjusted to cover various materials, shapes, and conditions of end fixity, modified by Lewis Gordon and by Rankine, became, under the title of "Rankine's formula," the standard for many decades.[38] It may be written thus:

$$p = \frac{f}{1 + \alpha\left(\frac{L}{g}\right)^2} \quad \dots\dots\dots \quad (5)$$

where p denotes the crippling "stress" $= \frac{P}{A}$

f ,, the stress at which the material will begin to crush
L ,, the length of the column
g ,, the radius of gyration of the column section.

Hodgkinson and Rankine considered only the case of a column centrally loaded. A formula of the same type is derived, however, if the load is eccentric, but α in equation (5) is then no longer constant, but can be shown (see Appendix) to have the value:

$$\alpha = \frac{\Delta n}{L^2} \quad \dots\dots\dots \quad (6)$$

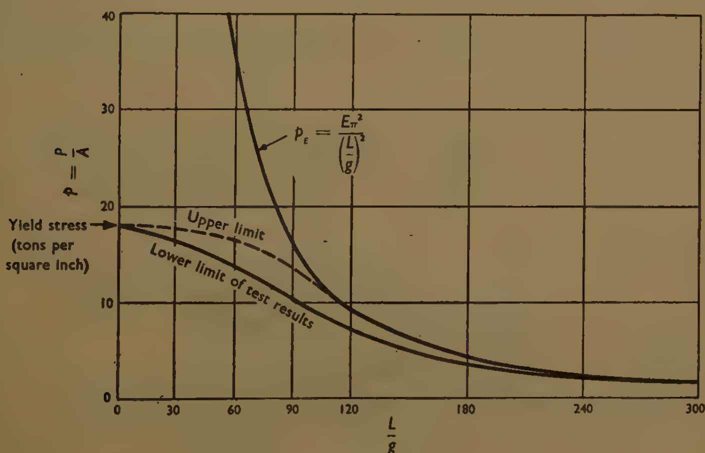
where Δ denotes the deflexion at which buckling commences
and n denotes the distance of the centroid of the section from the concave flange.

Many column formulae recommended by later authorities have been derived in this way. When applied to the calculation of the crippling load on a column intended to be concentrically loaded, these formulae differ mainly in the value allotted to Δ to suit some hypothetical assumption as to the probable deviation from the ideal conditions assumed by Euler. In his book published in 1887, Claxton Fidler included the deflexion which would be produced by an axial load if E varied in a hypothetical way across the section.[39] J. M. Moncrieff (1910)[40] argued that eccentricity would

be roughly proportional to $\frac{L}{g}$. Robertson,[41] writing in 1925, allowed an eccentricity at mid-span corresponding to a probable accidental initial curvature, which gave Δ the form already ascribed to Young and Perry and quoted in equation (2*a*).

In most column formulæ Δ is a function of p, such that the evaluation of p requires the solution of a quadratic equation; hence the surd form in which the final result is commonly expressed. There are other column formulae which are quite empirical. Since the final object of any column formula is to compute and tabulate values of p which will fit the lower limit of the test results plotted in *Fig. 4*, one is inclined to wonder why

Fig. 4



COMPRESSION FAILURE OF MILD-STEEL COLUMNS

tables should not be made direct from the graphs without so much mathematics and such protracted juggling with hypothetical variables. The lengths to which this subject has been stretched can well be seen by a perusal of the text and the full bibliography in E. H. Salmon's standard work on columns.[42]

The early cast-iron bridges, for the most part, took the form of arches. In the eastern states of the U.S.A., where a network of roads was developed in the early decades of the nineteenth century, facilities for casting heavy ribs were not available, and in Pennsylvania, James Finley (1762–1828) overcame this difficulty by building suspension bridges in which the deck was stiffened with girders.[43] After 1820 this system was adopted in England, where it was studied by C. L. M. Navier (1785–1836) of the Ponts et Chaussées, who toured Great Britain for the purpose and in 1823 expounded the theory of their construction in a memoir.[44]

Navier collected and published the Papers of his uncle, E. M. Gauthey, on bridges and canals, and in 1830 issued a new edition of Belidor's "*La Science des Ingénieurs*" which he brought up-to-date by means of footnotes. In the lectures which he prepared for students at the Ecole des Ponts et Chaussées and which were eventually published in 1826, he expounded a theory of structures embodying the fullest treatment so far of eccentrically-loaded columns, arches, and suspension bridges, and completed Coulomb's theory of beams by showing how to calculate deflexions by double integration of the moments—the method still given in modern textbooks.[45]

Navier's work on elasticity was focal and fundamental. As distinct from the piecemeal attacks made by his predecessors on isolated problems, Navier formulated a general mathematical theory of elasticity.[46] This was extended and modified by the work of many famous engineers and mathematicians, mostly Frenchmen, as shown in Table 1. No reference is made in Table 1 to work on elastic vibrations; this has a long history of

TABLE 1.—PIONEERS IN THE THEORY OF ELASTICITY (UNTIL 1860)

Name	Date	Principle contribution
R. Hooke	1678	Hooke's Law
Jakob Bernoulli	1705	The Elastic Curve
Johann Bernoulli	1717	Principle of Virtual Work
L. Euler	1757 1771	Buckling load on a column The shell as an assembly of bars
C. A. Coulomb	1784	Torsion in cylinders
Jacques Bernoulli	1789	The shell as a grid of bars
T. Young	1807	Young's Modulus Position of neutral axis in a column Columns with initial curvature
C. L. M. Navier	1820 1821 1826	Bending of plates Mathematical theory of elasticity General theory of structures, including deflexion of beams
S. Germain	1821	Curved plates
A. L. Cauchy	1822	Modified elasticity equations
S. D. Poisson	1829	Poisson's Ratio
G. Lamé	1833	Ellipsoid of stress Thick cylinders Maximum-stress theory of failure
with E. Clapeyron		Equality of internal and external work
G. Green	1837	Strain-energy equations
G. G. Stokes	1845	Modulus of Rigidity
Barré de Saint-Venant	1855	Analysis of combined stresses Maximum-strain theory of failure
E. Clapeyron	1857	Theorem of Three Moments

its own, but at the time it had no effect on the practice of engineering. By 1860, it may be said, tools had been devised to determine the stresses and elastic deformations in almost any structural member, but it took several further decades of experience in the use of these tools before they were reduced to forms suitable for use in the drawing office.

The Problem of the Jointed Frame

While the strength and elasticity of structural members were being studied, first by the use of the testing machine and later by the application of mathematics, the statical stability of roof trusses and open-frame bridge-girders was also receiving attention. Something approaching triangulated-truss construction was occasionally adopted by master masons in the middle ages, and illustrations of such trusses appear in a book by the Italian architect, Andrea Palladio (1518–1580).[47] But the anchorage for a tension member in timber is either clumsy or unreliable (if not both) unless some metal strapping and bolting is used. A roof framework of strutted beams, or of some kind of arch construction, could be made to span between the walls of a large hall without the use of iron. With the development in the late eighteenth century of the puddling process, however, wrought iron became available as a building material, for ties and bolts for instance, even when wood or cast iron was used for compression members. This composite construction was widely used both for roofs and bridges. The screwed bolt only replaced the pin and cotter in the early nineteenth century, when stocks and dies became normal equipment in engineers' workshops. There was, however, neither theory nor tradition to guide the designer of structural ironwork, and some of the novel constructions attempted during the rapid development of railways in the eighteen-forties failed, causing loss of life.

The consequent alarm led to the appointment in 1848 of a Royal Commission to enquire into the application of iron to railway structures. Many experiments were made with fluctuating loads applied in the laboratory, and some with locomotives on existing bridges. An attempt was made to establish a mathematical equation which would give the design load for a bridge, taking into account the weight and speed of the moving load and the inertia of the structure, but the task proved insuperable. The Commissioners advised in general terms that acceptance should be based on proof loads rather than on a specified mix of iron, and that dynamic loads should never be neglected in bridges of a span less than 40 feet. Where head-room permitted, the use of an arch was advocated; for low bridges, the hog-backed girder of cast iron, with a solid web and a broad bottom flange, was recommended. The art of building railway bridges was not in their view so settled that an engineer could with confidence apply general principles to unusual forms of construction such as open-framed or lattice girders.[48]

The use of timber trusses had meanwhile been developed on a wide scale in the U.S.A. Much ingenuity was displayed, and several failures had resulted, before Squire Whipple (in 1847) and Herman Haupt (in 1851) both published books in the U.S.A. on bridge building. In 1850, Colonel William Bindon Blood submitted a short paper to the Institution of Civil Engineers, and in 1851 Robert Henry Bow published his work on bracing.[49] All these authors resolved the forces in the members and studied the forces acting at each joint as a separate system of forces in equilibrium. The method of sections was introduced by August Ritter in 1863.[50]

This development of theory came at a most appropriate time. Robert Stephenson (1806–1869) had just completed, in 1849, his great High Level Bridge at Newcastle with tied arches of cast iron, on masonry piers at centres of 139 feet; and in 1850 the Britannia Tubular Bridge, of box section, constructed with wrought-iron plates and angles, and having 460-foot main spans. For this bridge many tests were made, on materials and on model girders, by Fairbairn and Hodgkinson. I. K. Brunel (1806–1859) at once adopted the new trussed girder, with curved compression booms in the form of cylindrical wrought-iron tubes, for his bridges at Chepstow (1852) and Saltash (1859). To carry out even a few works of this magnitude—not forgetting the vast number of works on a smaller scale—the leading engineers had to employ large staffs of assistants both in the field and in the drawing office and yet can have had little time to spare for their theoretical training.

Even before the turn of the century, Professor John Robison (1739–1805), a personal friend of James Watt and John Rennie, had included Mechanical Philosophy in his course on Natural Philosophy at Edinburgh University, but Rennie and Thomas Telford had both found it necessary to learn French and German to enable them to refer to the sources such as Belidor and other Continental writers. In 1838, a school of engineering was formed in King's College, London; and in 1841 a chair of engineering was founded in University College, London. In 1854, a Regius Professor of Engineering was appointed to Glasgow University. This chair was occupied for one year by Lewis Gordon and thereafter for many years by W. J. MacQuorn Rankine (1820–1872). Rankine did for British engineers what Belidor (a century before) and de Prony and Navier (more recently) had done for the French: he compiled a comprehensive series of text-books which, though containing little original fundamental matter except in the field of thermodynamics, embodied nearly everything of importance hitherto published on engineering theory. These books retained their place as standard works until well into the twentieth century.

Rankine added to beam theory the distribution of sheer stress across the section.[51] In treating jointed framework, he made use of a few simple reciprocal diagrams in which the forces in all the members of a jointed frame were set out in diagram, a method generalized by James Clerk Maxwell (1831–1879).[52] Maxwell's work was, however, couched in

mathematical terms, his notation was awkward and his description obscure. His reciprocal diagrams were of little use to engineers until their notation had been clarified by Bow.[53] The outstanding advances in the field of graphic methods of computation were, however, mainly the work of Continental geometers.

Graphic Statics

La Hire's prototype link polygon (1695) has already been mentioned, as well as Couplet's extension of its use (1726). Belidor made use of the triangle and the parallelogram of forces. Prony developed a graphical solution for earth pressures, based on Coulomb's wedge theory. Gaspard Monge (1746–1818), Professor of Stereometry at the Ecole Polytechnique, introduced the methods of projective geometry and J. V. Poncelet developed them and applied them to mechanical problems; but each problem was solved separately and the basic system of treatment was difficult.

Carl Culmann (1821–1881) studied at the Polytechnic at Carlsruhe, where he gave particular attention to projective geometry, which he applied to the design of bridges in his subsequent practice of railway engineering. Between 1849 and 1851 he visited England and North America, where he took a particular interest in open-frame bridges and the work of Squire Whipple. The notes he published on his return to Europe stimulated interest there in the American type of bridge construction. In 1859, Culmann was appointed Professor of Engineering at the newly established Federal Polytechnic at Zurich, where he taught and considerably expanded the application of known methods of graphical computation. In 1864, he published the first edition of his work on graphic statics, in which he generalized the use of force and link polygons, and applied them to the determination of bending moments, moments of inertia, and other properties of sections, and to the general solution of forces in framework.[54]

Wilhelm Ritter (1847–1906) studied under Culmann, served at one time (1869–73) as his assistant, and in 1882 succeeded to his chair of engineering. Ritter collected Culmann's unpublished papers and issued their substance with additional material of his own, thus making graphic statics a practically complete subject of study in its own right.[55]

Luigi Cremona (1830–1903) also devised a general system of graphical computation, set out in such a way that it could be applied without previous study of the projective geometry on which it was based. Cremona's book on graphic statics, published in 1872, has been translated into English by Professor Hudson Beare.[56]

In France, the methods of both Ritter and Cremona became known through the writings of Maurice Levy (1838–1910).[57] Others have followed the pioneers, most notably Otto Mohr (1835–1918), Professor at the Technischen Hochschule in Dresden, and H. Müller-Breslau (1851–1925), who was Professor at the Technischen Hochschule in Berlin.

Mohr's contributions are scattered through numerous books and articles

in journals, but he collected and re-edited the subject matter of his most important papers in his "*Technischen Mechanik.*"[58] His most useful innovations were his well-known stress circle, his improvement on Williot's graphical determination of the deflexions in a framework, and his well-known "area-moment" method of finding the deflexion in a beam. This method was further developed by Müller-Breslau, whose last work probably contains the most compendious and generalized treatment of the whole subject of graphic statics.[59]

The influence line, a device first described by E. Winkler in 1868, has proved of immense value in the graphical solution of problems in structures subject to moving loads. The use of the influence line has been extended to cover the distortion of continuous girders and stiff-jointed frames by applying Clerk Maxwell's Law of Reciprocal Deflexions (1864), which was generalized by E. Betti in 1872.[60] In its simple form this law may be stated thus: A and B are any two points in an elastic body or structure; a force U at A produces a deflexion b at B while a force W at B produces a deflexion a at A. Then, the movements a and b being measured respectively along the lines of action of U and W:

$$Ua = Wb \quad \ldots\ldots\ldots \quad (7)$$

or for any load W:

$$a = \frac{W}{U} b \quad \ldots\ldots\ldots \quad (8)$$

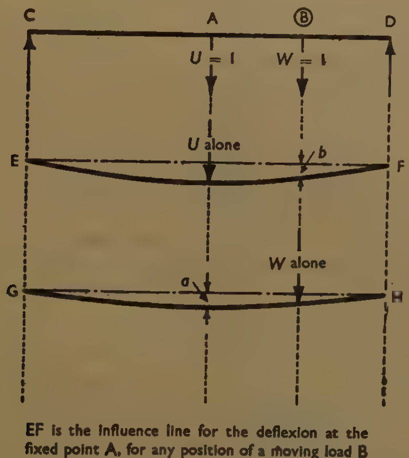
By making U and W both unit forces, Müller-Breslau showed that, if A is a fixed point and B movable, the graph of b is the influence line for deflexions at A produced by the load at the movable point of application B. (See *Fig. 5.*)

The main contributions to graphic statics, some of which cannot be dealt with in detail in this Paper, are shown in Table 2.

American practice in framed-girder construction was, for a long time to use actual pins at the node points; British and European practice was to form rigid joints, riveted or bolted, but still to calculate the forces in the members as though the joints were pinned. Some secondary stresse were thereby introduced. In the eighteen-eighties these were investigated by strain-energy methods. In heavy bridge construction, the effect o secondary stresses was found to be slight, but in modern very light frame work this is by no means so unless great care is taken to avoid eccentricit of loading at connexions. Graphical methods proved of immediate valu in the drawing office, particularly to engineers and draughtsmen who coul more readily grasp the meaning of a diagram than that of a mathematica equation. Unfortunately, college courses in engineering have tended i recent decades to over-emphasize mathematical methods and to neglec graphical alternatives. Mathematical methods are easier than graphica methods to teach and to examine: in the hands of a competent mathe matician they provide a solution in a shorter time; but they do not giv

the visually-minded person so clear a grasp of what is actually happening. The greater apparent accuracy of a calculated result is also frequently spurious, since the engineering data selected for the purpose of calculations

Fig. 5



INFLUENCE LINE FOR DEFLEXION FROM MAXWELL'S LAW OF RECIPROCAL DEFLEXIONS

TABLE 2—GRAPHIC STATICS

Name	Date	Main contribution
. de La Hire	1695	Force and link polygons Arch with frictionless voussoirs
. Couplet	1729	Extension of La Hire's work
. McQ. Rankine	1857	Force and link polygons applied to framed structures
. A. D. Ritter	1863	Method of sections
Clerk Maxwell	1864	Reciprocal diagrams
Culmann	1866	Graphic statics
. Winkler	1867	Influence lines
. C. Mohr	From 1868	Stress circle Beam deflexions by area-moments Generalized reciprocal diagrams
. H. Bow	1873	Simple notation for reciprocal diagrams
illiot	1877	Displacement diagrams (improved by Mohr)
. T. Eddy	1878	Graphic solution of strain-energy problems
Claxton Fidler	1887	Continuous beams: characteristic point method
. Ritter	From 1888	Completion of Culmann's work
Perry	1897	Graphic solution of the rigid arch
Ostenfeld	1905	Extension of use of characteristic points
. F. B. Müller-Breslau	1927	General treatment of the whole subject

are never complete and seldom warrant great exactitude in computation ; it is also easier—without knowing it—to make gross errors in calculations than in the use of graphics.

Design Methods Based on Strain Energy

The methods of graphic statics are, in general, limited to the solution of problems in which the balance of forces is not seriously affected either by the distortions of individual members, or by the stiffness of members and their connexions. If a framework, virtually pin-jointed, has " redundant " members, additional to those just sufficient for complete triangulation, the static balance of internal forces is affected by the relative distortion of the members. If members are connected by stiff joints, bending forces are introduced which may be of greater importance than the direct forces. The key to the solution of the forces both in redundant and in rigid frameworks was found in the application of the two principles of virtual work and of least work. According to the principle of virtual work, the first application of which to a structural problem was published in 1833, by Lamé, who attributed it to Clapeyron, the energy stored in a frame of which the members deform in the manner of springs will be equal to the work done by the external forces when their points of application move as the frame distorts.[61] According to the principle of least work, the distribution of the forces in the members of the frame will be such that the total energy stored in these springs is the minimum. The main steps in the development of strain-energy theory are shown in Table 3.

Following preliminary work by Maxwell, Mohr, and Menabrea between 1850 and 1870, A. Castigliano (1847–1884) systematically applied the method of least work to a wide range of examples in his classic work published in 1879.[62] The method of least work was further developed and applied by Betti, Mohr, Müller-Breslau, and others. It was also in 1879 that Manderla analysed the secondary stresses in a truss with stiff joints by the slope-deflexion method, that is, by treating the angles of rotation of the joints as the unknowns to be solved, rather than the moments and the direct forces. A fully documented historical account of later work on secondary stresses in bridges was given in November 1924, to the American Society of Civil Engineers, by C. V. von Abo.[63] The slope-deflexion method was first applied to determining the moments in a stiff-jointed frame by Wilson and Maney in 1915.[64] A. Ostenfeld obtained equations similar to those of Maxwell and Mohr, except that he treated deformations and not the loads as the unknowns.[65]

In nearly all the methods of calculation which depend on strain energy it is assumed that the material will deform in accordance with Hooke's Law and that the working stresses for which the members are designed are well within the limit of proportionality, whether the stresses are simple or complex. For complex or fluctuating stresses under combined bendin

TABLE 3.—APPLICATIONS OF STRAIN ENERGY

Name	Date	Principal contribution
Jakob Bernoulli	1705	Elastic curve of beam
L. Euler	1757	Elastic curve of column
G. Lamé	1833	Principle of Virtual Work applied to structures
E. Clapeyron	1857	Theorem of Three Moments
L. F. Menabrea	1858	Least Work applied to trusses
J. Clerk Maxwell	1864	Law of Reciprocal Deflexions
E. Betti	1872	General equation between external work and strain energy
Williot	1877	Deflexion of a statically determinate frame
A. Castigliano	1879	Equilibrium of elastic systems
H. Manderla	1879	Secondary stresses in rigid frames by slope-deflexion method
O. C. Mohr	1868 onwards	Area-moment method of finding deflexions Clarification of Maxwell's and Williot's work
H. Müller-Breslau	Various dates	Extension of Maxwell's and Mohr's work on indeterminate structures Deflected load-line as influence line
H. Müller-Breslau	1885	Truss deflexions
T. Claxton Fidler	1887	Characteristic-point method for moments in continuous beams
A. Ostenfeld	1905	Development of characteristic-points
E. G. Coker	1911	Photo-elasticity
W. M. Wilson and G. A. Maney	1915	Analysis of rigid frames by slope-deflexion method of analysis
G. E. Beggs	1927	Slopes and deflexions by models
Hardy Cross	1929	Moment-distribution method of finding moments in rigid frames

and torsion, the assessment of the margin of safety is not always a simple matter, and opinions have differed as to the basis on which the limit of safe stress should be computed. Lamé assumed that safety was assured if the maximum principal stress did not exceed the normal working stress; in this he was followed by Rankine, although Saint-Venant had favoured the maximum strain as the criterion.[66] When lateral distortions, calculated in terms of Poisson's Ratio, are included, the two criteria are not identical. For brittle materials generally, and for ductile materials in which the principal stresses are both tensile or both compressive, the tendency to fail where principal stress occurs is borne out by tests to destruction; but for ductile materials, subjected to more complicated systems of stress, greater accuracy and generality have been claimed for one or other of the strain-energy theories of failure shown in Table 4.[67]

Since the beginning of the century, model analogies have been developed as a means of measuring deformation under complex stress where computation is difficult, or where the theory is suspect. One method is to observe the movement and distribution of colour-fringes when polarized light is passed through a celluloid model shaped and strained as in some part of a structure or mechanism. This method has been applied by E. G. Coker

TABLE 4.—THEORIES OF FAILURE

Criterion	Authority	Date
Maximum stress	G. Lamé	1852
	W. L. M. Rankine	
Maximum strain	B. de Saint-Venant	1855
Maximum shear stress	C. A. Coulomb	1773
	H. Tresca	1868
	G. H. Darwin	1882
	J. J. Guest	1900
Internal friction	C. A. Coulomb	1773
	C. L. M. Navier	1833
	J. Perry	1897
Modified maximum shear	O. C. Mohr	1900
Maximum strain energy	E. Beltrami	1885
	B. P. Haigh	1919
Maximum shear-strain-energy	M. T. Huber	1904
	R. von Mises	1913
	H. Hencky	1924

and subsequently by others to a wide range of problems in localize stress.[68] A method applicable to torsion problems is to study the shap assumed by a convex soap-film, slightly distended by air pressure, which i made to cover an aperture cut to the shape of the perimeter of the torsio member. L. Prandtl showed, in 1903, that the equations for the torsiona stress in the member and for certain properties of the solid figure of th soap-film were analogous. The practical application of this analogy ha been developed by A. A. Griffith and G. I. Taylor.[69]

PRESENT-DAY DEVELOPMENTS

Before the close of the nineteenth century, methods of solving even th most complicated problems had been devised in principle, but their applica tion in detail was often extremely laborious. All the strain-energy methoc involve the calculation and summation of the effect of every load on ever member of a continuous frame. If the frame comprises many member the process of computation is long, tedious, and expensive. The sectio properties of every member must be known, or assumed, before the wor can begin, and if the result shows that some of the sections assumed ai not economical, the work must be done all over again—perhaps more tha once for each of several systems of loading. This would be onerous eve if the result was fully reliable; but when a satisfactory solution has bee found on paper, the computed stresses or moments may differ considerab from those developed in the actual structure should even small errors b made in setting-out or in workmanship, or should slight differential settl ment take place. It is therefore natural that, in the design of buildir frames in riveted or bolted steelwork, empirical methods in which distortic is ignored have continued to be in favour with designers.

In frames of reinforced concrete or welded steelwork, however, the neglect of stiff-jointing cannot be tolerated, and a demand for methods of analysis and check that are less wasteful of time and labour than a full strain-energy analysis has become insistent. Within the past few decades, this demand has been met in several ways: by the use of calculating machines to speed up the process of computation, and to prevent numerical errors; by arriving at the actual stresses or moments by methods of successive approximation; and by tests on models, or on full-scale structures fitted with strain-gauges.

Many types of calculating machines, mechanical and (more recently) electronic, have been devised, and the latest of these make short work of the solution of simultaneous equations with many unknowns. Engineers are not likely to credit machines, however ingenious and elaborate, with miraculous powers of thinking for themselves; the thinking part has been done in advance by the designer of the machine. The best designed and most skilfully constructed machine—when in perfect order—can only do what its designer has planned it to do, and no mean skill in mathematics is still required from an engineer, who must feed his problems into the machine in a form which it can digest.

Another way to reduce the labour of computation is to obtain a result by means of successive approximation, carried to a degree of accuracy comparable with that of the loading and other data. For building-frames, for instance, the moment-distribution method introduced by Professor Hardy Cross has proved most valuable.[70] This method, starting from an assumption of complete fixity at every joint, and proceeding by alternate balancing and distribution of the moments, is too well known for it to need elaboration here. It has been followed by more general Relaxation Methods first applied by L. K. Richardson to the stresses in a masonry dam and later generalized by R. V. (now Sir Richard) Southwell.[71]

An interesting example of the choice of method for an intricate design was described by Gilbert Roberts in a Paper which he recently presented to the Institution of Civil Engineers, on the structural design of the Dome of Discovery for the 1951 Festival of Britain.[72] The Dome had a treble intersecting system of ribs springing from a ring girder, with 37 internal nodal points each with 6 degrees of freedom, and 24 boundary nodal points each with 4 degrees of freedom, making a total of 318 elastic unknowns—a problem too formidable for even the most elaborate computing machine. Manufacture had to proceed as fast as drawings could be made available. Each rib was therefore designed as a two-hinged arch to carry its own proportion of roof load. This preliminary design method was expected to give an over-estimate of bending stresses but an under-estimate of some direct thrusts; and so it proved when an accurate calculation was performed by Mr T. O. Lazarides, using relaxation methods, and when tests were applied to the erected structure. It was doubted whether the work of computation could have been simplified if an electronic

calculator had been available, because the main part of that work consisted in the preparation of the relaxation data. Calculations by relaxation methods, the Author understands, do not take a form suitable for mechanical computation.

The best check on the behaviour of any structure under load, since it gets rid of all the hypothetical assumptions made in the calculations, is a test on the completed structure; but at the design stage the nearest practicable substitute for a full-scale test is a test on a model that is constructed, and modified or remade as necessary, so that its parts deform in the same ratio as those of the intended structure. For frames in one plane, an ingenious technique was developed by G. E. Beggs of Princeton University, and first described by him in 1922.[73] A xylonite model of the framework is made with profiles so cut that the stiffness of each member in the model is proportional to that of the corresponding member in the structure. A cut is made, and a deliberate movement a is imposed, at some selected point A, such as the base of a main column, and consequent movements b at other points B are read by means of a microscope. Influence lines can thus be drawn, in accordance with Maxwell's law of reciprocal displacements, and the force evoked at A by the load applied at B can be calculated from equation (7). An alternative method of investigating the stresses in a plane frame represented by a xylonite model (as mentioned previously) is to observe under polarized light the colour-fringes formed when the frame is deliberately distorted.

Distortions observed in a model can be used to measure stresses in structures which would defy analysis by any other method. For instance in a recent Paper, S. C. Redshaw and P. J. Palmer described the investigation of the behaviour of a Delta aircraft, when flying at high speed at a great altitude, by means of a three-dimensional xylonite model with scale factors so selected that the model could be used to test stiffness, strain distribution, resonance, and flutter. Special test instruments had to be devised for use on the bench and in the wind-tunnel. Full-scale tests made later on the prototype, duly adjusted in the light of the tests on the model, confirmed the conclusions reached by the model-tests.[74] A similar model technique, but less elaborate, is being applied to the deformation of shell roofs under various combinations of loading. Full-scale steel-framed pylons for the electricity supply grid have been tested to incipient failure under measured loads applied to represent cable-, wind-, and snow-loads and the effect of broken wires. The rig involves the use of wire-rope winches, lofty anchor-towers, and a large number of strain gauges. Strain gauge measurements have been greatly facilitated by the modern small resistance gauge which can be fixed to the structure with adhesive.

Another modern development is the study of the re-distribution of stress in the elements of a structure, when material in a certain member or connexion is stressed beyond its limit of proportionality. Much theoretical work on plastic deformations was carried out by German engineers in th

early decades of the present century, and a promising effort by a British team to devise a design method based on plastic deformations is the subject of much interest and considerable controversy at the present time.

Conclusion

It has not been possible within the scope of a single Paper to include every development of importance in the historical development of structural theory. Nothing has been said, for instance, of reinforced-concrete theory, soil mechanics, and pre-stressing. The impact of improved technique in testing has received but passing reference. These omissions are regrettable, but inevitable. It has been possible only to follow the main line of development from the scientific revolution of the seventeenth century, through the urge of practical necessity at various stages, to the problems which face the designer of today. The study of engineering history, or of any other branch of history, is justified only if it links the past, the present, and the future into one chain of cause and effect, no part of which is significant in isolation. Engineering design is an art, and was so practised with no mean success before any theory was developed. Theory is a tool of great power in the hands of the artist, but may be a two-edged weapon in the hands of the unimaginative practitioner. It is derived to meet a recognized need, and requires continual adjustment to meet that need efficiently. Periods of elaboration and simplification tend to alternate. A perusal of the transactions of engineering societies in the past half-century suggests that we have gone far along the road of elaboration, but the modern reconsideration of the structure as a whole, full-scale or in model, deforming elastically or plastically, may shortly lead us forward to a new simplification, on a plane of closer correspondence between intention and performance than was possible with the simple theories of a hundred or even fifty years ago.

Acknowledgements

The Author wishes to state that he is indebted both to the Director of Building Research for permission to undertake the compilation of the Paper, and to his colleagues who have helped with information and constructive criticism.

The Paper is accompanied by five sheets of diagrams, from which the Figures in the text have been prepared, and by the following Appendix.

Appendix

Note on the Theory of Columns

Loading concentric

p denotes the actual failure load per unit of column section $\frac{P}{A}$

p_E denotes Euler's crippling load per unit area, which from equation (1) has the value:

$$p_E = \frac{P_E}{A} = \frac{EAg^2\pi^2}{AL^2} = \frac{E\pi^2}{\left(\frac{L}{g}\right)^2} \quad \dots \dots \dots \quad (9)$$

f denotes the extreme fibre stress at which crushing will commence.

Then if p denotes the harmonic mean between p_E and f:

$$\frac{1}{p} = \frac{1}{p_E} + \frac{1}{f} = \frac{p_E + f}{p_E f} = \frac{1 + \frac{f}{p_E}}{f}$$

Therefore:
$$p = \frac{f}{1 + \frac{f}{p_E}} = \frac{f}{1 + \frac{f}{E\pi^2}\left(\frac{L}{g}\right)^2} = \frac{f}{1 + \alpha\left(\frac{L}{g}\right)^2} \quad \dots \dots \quad (10)$$

Rankine used this form, but obtained α empirically.

Loading eccentric by a distance Δ from the central axis

$$f = \frac{P}{A} + \frac{M}{Z} \quad \dots \dots \dots \dots \quad (11)$$

where M denotes bending moment $= P\Delta$

Z „ modulus of section $= \frac{I}{n} = \frac{Ag^2}{n}$

Therefore:
$$f = \frac{P}{A}\left(1 + \frac{\Delta n}{g^2}\right) = p\left(1 + \frac{\Delta n}{g^2}\right)$$

and:
$$p = \frac{f}{1 + \frac{\Delta n}{g^2}} = \frac{f}{1 + \frac{\Delta n}{L^2}\left(\frac{L}{g}\right)^2} = \frac{f}{1 + \alpha\left(\frac{L}{g}\right)^2} \quad \dots \dots \quad (12)$$

In equation (10), α is a constant which depends on the physical properties, f and E, of the material of the column. In equation (12), α is a variable which depends on the dimensions of the column and on Δ which is a function of P and therefore of p. The determination of p usually requires the solution of a quadratic equation.

For instance, the Perry formula depends upon the substitution in the first expression for p in equation (12), of Δ as given by equation (2*a*), thus:

$$p = \frac{f}{1 + \frac{\Delta n}{g^2}} = \frac{f}{1 + \frac{en}{g^2}\left(\frac{p_E}{p_E - p}\right)} \quad \dots \dots \dots \quad (13a)$$

Rearranged, this equation takes the form:

$$p(p_E - p) + \frac{en}{g^2} p_E p = f(p_E - p) \quad \dots \dots \dots \quad (13b)$$

which is a quadratic equation in terms of p and e as the only unknown quantities. Robertson found that the graph of p calculated from this formula could be made to fit the lower limit of the test results plotted as shown in *Fig. 4*, if $\frac{en}{g^2}$ was given the value $0{\cdot}003\frac{L}{g}$. This means that e, the accidental deviation of the centre-line of a

reputedly straight column from the axis of loading, is in practice roughly proportional to the slenderness ratio.

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Discussion

The Author introduced the Paper with the aid of a series of lantern-slides.

Dr A. W. Skempton said that it had seemed to him for many years that insufficient attention was paid to the history of civil engineering. The Author had mentioned six previous Papers or articles touching on that subject, published within the past 60 years, only three of which were by British authors, and only one of those by an engineer. Structural theory obviously played a basic part in civil engineering and there should surely be more Papers on that subject. But it was, after all, only one of the fundamental engineering sciences, and there were probably few engineers who had more than a casual knowledge of the history of hydraulics, for example, or of the history of surveying or soil mechanics.

Those were, however, the theoretical aspects of civil engineering, and it might be expected that more information was available on the practical side. But there was, in fact, a rather marked lack of such information and a corresponding degree of ignorance among civil engineers of the history of their own profession. Was it right that such ignorance should exist? If engineers were not sufficiently interested to discuss their own history, and perhaps occasionally to prepare a Paper on some historical point, it

was not surprising if others were not interested in their work. A certain amount of attention was rightly being paid to publicity at the present time, and if that were linked to the history of the civil engineering profession it would, in the long run, pay good dividends. The informed public of to-day were historically minded to an unprecedented extent, and tended to judge values in the light of historical perspective.

Although civil engineering science was not, in effect, more than about 150 years old, civil engineering practice had one of the longest and the most honourable records of any of the professions, and was comparable in that respect perhaps only with law, medicine, and architecture. There were many excellent books dealing with the history of those professions, ranging from short popular expositions to works of scholarship on particular periods or particular men. Yet how many books could be mentioned which rendered the same service for civil engineering? The answer was that there were very few indeed, and of those few some were almost unobtainable.

In that connexion, Dr Skempton had in mind a brilliant book on engineering in the Renaissance by Barclay Parsons,[75] a distinguished American engineer who had devoted the latter part of his life to original research on that subject in Italy and France. Parsons's work was outstanding and demonstrated beyond doubt that civil engineers had played an important part—and a part which had been fully recognized by their contemporaries—in developing the life and economy of the expanding civilization of the Renaissance. Yet very few copies of that book were available (although there was one in the Institution library) and, since Parsons's work was practically the only one giving any information on that vital period, it was not at all surprising that the tacit assumption was generally made that there had been little or no civil engineering in those days. On the other hand, there was, for example, a great number of books dealing with the architecture of that period, some of which were so well written and illustrated that they were often to be found on the bookshelves of educated people who were themselves not architects but who were interested in social, artistic, or economic history.

It had long been thought that the mediaeval cathedrals had been achieved more by wit and native ingenuity than by any other means, but recent researches by John Harvey[76] and others had proved that that view was, of course, incorrect. It was only necessary to go into almost any cathedral and witness the skill and knowledge necessarily involved in its design and construction to realize that error. The truth was that the cathedrals had been designed by highly trained men, combining a knowledge of both architecture and engineering, and with a social status and salary well above that of the skilled mason. There could be no doubt that those

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[75] W. Barclay Parsons, "Engineers and Engineering in the Renaissance." Baltimore, 1939.

[76] J. H. Harvey, "The Gothic World 1100–1600." Batsford, London, 1950.

men had been fine architects, and a study of the structures they had erected showed them also to have been fine engineers. In fact they had acted directly as engineers in the building of bridges—for example, Henry Yevele (*circa* 1320–1400) had designed Westminster Hall, the nave of Canterbury Cathedral, and Rochester Bridge.[77]

The Author had quoted a number of instances where those buildings had failed, but there was a far larger number of cases where they had stood up, and were still structurally sound, 400 to 800 years after construction. Present-day engineers should feel proud that they were, professionally speaking, descendants of those people who had been the designers of the most beautiful buildings and probably some of the most superb and daring structural engineering work to be found in the world.

Dr Skempton then referred to early river navigation in England—another subject on which there was a dearth of historical information. The canals had really opened up commercial transport and made possible the full development of the Industrial Revolution, but the first canals had not been built in England until about 1760, and then in response to a demand which had been steadily growing for about a century. Industrial progress in the hundred years following the Restoration had been facilitated to a large extent by river transport. In 1660 there had been 700 miles of navigable inland waterways, almost all of which had been natural rivers. By 1725, when Daniel Defoe had carried out his investigations [78] into the commercial structure of Great Britain, the mileage had been increased to nearly 1,200, and practically all of the additional 500 miles had been made navigable by the civil engineers of those times. Those men had laid the foundations of a heavy-transport system in Great Britain, and it was from them that the canal engineers had learned much of their art. Yet practically nothing was known of those engineers of the late 17th and early 18th centuries, and most of what was known was only to be found in one Ph.D. thesis.[79]

It was clear, then, that a great deal of work would have to be done before much of the long history of civil engineering became accessible. Research work such as that of Barclay Parsons and the Author of the present Paper was vital to an understanding of the great contributions of the engineering profession to society, both present and past. Dr Skempton also emphasized the need for articles, books, and lectures to be prepared for a wider audience, including university students of engineering. A book recently published in Switzerland [80] was an example of what he had in mind. In the introduction to his book [81] on the origins of modern science,

[77] J. H. Harvey, " Henry Yevele." Batsford, London, 1944.

[78] Daniel Defoe, " A Tour through the Whole Island of Great Britain." London, 724–26.

[79] T. S. Willan, " River Navigation in England, 1600–1750." Oxford, 1936.

[80] Hans Straub, " *Die Geschichte der Bauingenieurkunst* " (" The History of the Art of Constructional Engineering "). Basle, 1950.

[81] " The Origins of Modern Science, 1300–1800." Bell & Sons, London, 1949.

Professor Butterfield had observed that it was becoming realized that knowledge of the history of science was of value in a liberal education—and that was no less true in the case of civil engineering.

Many conclusions could be drawn from the study of history, but two stood out very forcibly. The first was that, in any great period, the designers had used to the utmost limit the technical resources available to them. That was evidently true in the mediaeval cathedrals, and had also applied in the time of Telford and Rennie, when metal structures were being built for the first time. To build the Menai Bridge, Telford himself had had to advance the knowledge of the properties of wrought iron and the theory of suspension bridges; and the design of the Britannia Bridge by Robert Stephenson had involved a major piece of research—possibly the most important single research in structures of the whole of the 19th century.

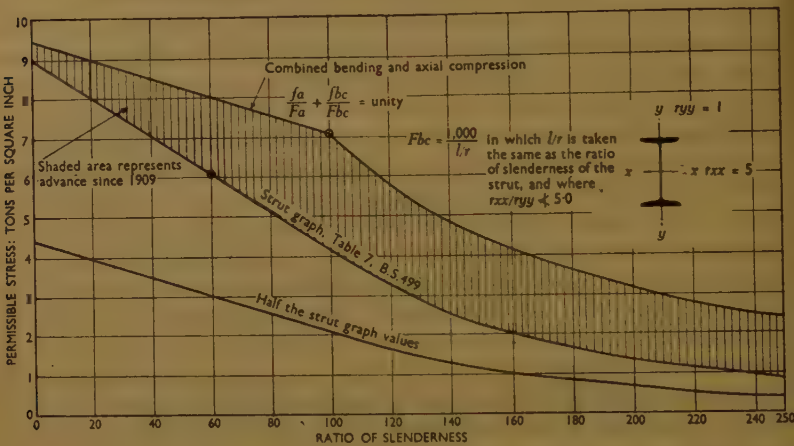
In the past 150 years, a new resource—structural theory—had been added to the equipment of the structural engineer. In the days of masonry construction, extending from the Roman period to the 18th century, the designer had had past experience to guide him, skill and daring to lead him forward, and, since he was also an architect, he had possessed a fine sense of proportion. At the present time, the designer still had past experience and should still have skill and daring; he also had structural theory, which had to be used to the fullest possible extent in design. But in acquiring that theory the engineer had temporarily or partially lost his sense of architecture and proportion. For exceptional men such as Robert Maillart that had not been so, and some of his reinforced-concrete bridges were masterpieces of design from all points of view. For most engineers, however, the second moral to be drawn from history was that, in striving to use theory to its extreme practical limit, they should not lose sight of the great tradition of aesthetic design in structural engineering. The task at the present time was in some ways a more difficult one than that facing the designers of older times but, if the challenge was met, a new great epoch in structural work would be born.

Mr G. A. Gardner pointed out that the subject under discussion was the historical development of structural theory and not that of engineering itself. It was a very important subject, not in the sentimental way, but in a practical way, because it was only by realizing how certain theories had been evolved that it was possible to judge present-day theories properly and, indeed, to legislate with regard to the future. Mr Gardner was of the opinion that all young engineers should be required to study the subject and to pass an examination in it.

Another important point was that practising engineers in general very seldom overcame that lag which tended to exist between what they practised and what the inquiring investigator had found out beforehand. A notable example of that was the case of struts subjected to cross-bending. Less than 40 years ago, the engineer had been forced by Regulations to limit the load per unit area on a strut, plus the stress due to cross-bending,

to the permissible pillar stress; the legislators having overlooked the work of Perry, Winkler, and many others who had solved the problem a long time before. In that connexion *Fig. 6* showed a graph of a strut subjected to bending and carrying half its permissible strut load; the hatched area indicated the progress which had been made since the Building Act of 1909.

Fig. 6



AMOUNT OF BENDING WHICH CAN BE ALLOWED IN A STRUT LOADED TO HALF ITS ALLOWABLE AXIAL LOAD

An interesting point on which the Author might possibly give som enlightenment was why it was that the approach of many early investigator had been philosophically untenable. For instance, it was difficult t understand Galilei's assumption that his beam fibres were in tension onl and equally stressed. Could it, perhaps, have been the result of a lack o basic philosophical training?

Mr Gardner then showed a slide of a portrait of Euler, which he alway kept in his room in order to inculcate a reverence upon young men comin up for oral examination on promotion and to keep in mind the great deb which engineers owed to Euler. Mr Gardner mentioned that, when h had obtained that print, he had discovered on the back the inscriptio "L. Euler; teacher of arithmetic"!

Euler's formula, as at present known, was deduced from the strut ben to a sine curve, and that had led to considerable confusion. That ha been well illustrated upon one occasion when a student had asked wh could make a strut bend if it were of isotropic material and loaded axially The point was that in the formula deduced by taking the sine curve an a constant chord length a state of indifferent deflexion could be deduce

which, indeed, was very nearly true in an "ideal" specimen; but it was to be recalled that Euler's value for the resilient force was for a deflexion approaching zero (mathematically).

Fig. 7 showed how very nearly the deflexion *was* indifferent in a slender timber strut loaded to Euler's value.

Apropos of the Euler complex generally, Mr Gardner recalled how the late Ernest George Beck used to be incensed at text-book illustrations showing the cap of a strut below its base as an extension of Eulerian deformation!

Those who were old enough to have practised strut theory 45 years ago would remember how elementary it had been, and Mr Gardner remarked how, as a young man, he had been struck by the elementary mathematical subtlety of Gordon Rankine's formula expanded from the reciprocals of a stout and a slender strut, as shown by the following equations.

Let $$\frac{1}{P} = \frac{1}{q \cdot P_s} + \frac{1}{q' \cdot P_l}$$

in which $$P_s = c \cdot A$$

and $$P_l = \frac{\pi^2 EI}{l^2} \text{ (Euler).}$$

Then $$P = \frac{(qc)A}{1 + \left\{\frac{qc}{q'\pi^2 E}\right\}\frac{l^2}{g^2}}$$

which written in its usual form is:

$$\frac{P}{A} = \frac{fc}{1 + a\left(\frac{l}{g}\right)^2}$$

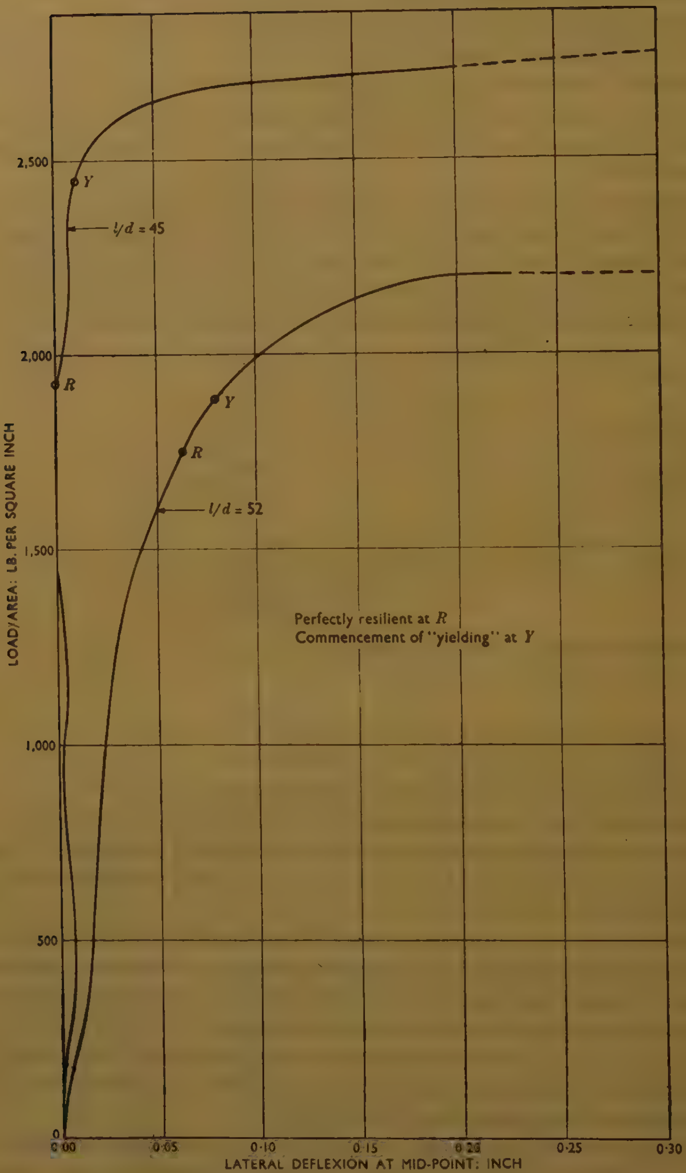
—similar to that given in the Author's text.

The Author had drawn attention to the danger of too much theory and t was worthy of note that there had been no less than seventy-six different estimates of the value of the initial eccentricity of a practical strut from 822 to 1919!

On the question of graphic statics Mr Gardner agreed with the Author hat the graphical method was very important, not so much because it rovided, perhaps, an easier solution, but because it was more visual. It as important in graphic statics that students should study its fundamental istoric development because it was commonly found that if the instructor ho was unfolding the magic diagram was asked what one of the lines really eant, he could not give a satisfactory answer.

The Author had mentioned some of the earlier bridges in the United tates, and Mr Gardner's last slide showed an example of a Whipple ridge of 1870, which was compounded of a number of simple primary usses. One rather shuddered to think how the bridge would undulate

Fig. 7



TYPICAL DEFLEXION OF "LONG" TIMBER STRUTS. (Air-seasoned Pitch Pine, flat ends.)

as a train load passed over each of the vertical struts and successively stretched the long tension rods of the numerous elemental trusses.

Mr O. Bondy observed that practising engineers might look upon a pursuit of historical interests as a luxury which they could ill afford. On the other hand, some information on the historical background of the development of their tools was no doubt very useful in everyday routine.

The Paper certainly supported the impression which Mr Bondy had gained over many years when comparing the literature on the subject in Great Britain with the literature from abroad, namely, that the same contributions to the development of structural theory were called by different names in Britain and abroad. To give one example, in the standard textbooks in Great Britain it was not easy to find mention of Cremona in connexion with reciprocal figures or force diagrams, whereas on the Continent they were invariably ascribed to Cremona. Another example was the modulus of elasticity: Mr Bondy assumed that the designation "Young's Modulus" came from the name of Dr Thomas Young, whose work was discussed in detail in the Paper, but he was not sure. The puzzling feature was that nearly 200 years ago Euler had developed his basic formula, and if Young's Modulus had been known to him then it had not been called by that name, because Dr Thomas Young had only published his works half a century later.

A third example arose in connexion with earth pressure. The Author had given one or two examples of earth-pressure theory development and had referred to Poncelet, but that name was not known to Mr Bondy in that connexion and it occurred to him that Poncelet's theory might be the same as that known by Rebhann's name on the Continent. Rebhann's graphical construction for deriving the intensity and the angle of the earth pressure to the horizontal was one of the favourites among students as well as among practising engineers.

Mr Bondy was in full sympathy with the Author's remarks concerning graphic statics, but admitted that he might be somewhat biased because graphic statics had been a separate subject in a $4\frac{1}{2}$-year degree course on civil engineering matters in Vienna. There had been a separate Chair for structural mechanics and graphic statics at that time, and the Professor in charge had been a Professor Anton Zschetzsche—a great character and quite outstanding in his original ways, which had attracted crowds of students. Perhaps that was one of the contributory reasons why graphic statics had become another favourite amongst civil engineering students at that period.

Dr Norman Davey said that, without bringing politics into the discussion, he wanted to refer to one phrase used by Mr Churchill in a speech which he had made in May 1945, when he had said "It is only from the past that we can judge the future." Mr Churchill had been referring, of course, to historical events, but his remarks were equally applicable to the science of engineering.

In his conclusion, the Author had stated that "The study of engineering history, or of any other branch of history, is justified only if it links the past, the present, and the future into one chain of cause and effect." Having read the Paper Dr Davey had tried to detect what those links were and whether there was, in fact, any continuity. It seemed that six or seven hundred years ago, in the so-called Gothic period—which he believed had been the age of great experiments—there had been men who, by long experience and with a good deal of intuition, had been able to balance small stones one upon the other. In the 16th century there had certainly been some attention paid to the laws of static equilibrium. And towards the latter end of the Renaissance there had been a thorough understanding of dome and arch construction, which could be called engineering as that term was now understood.

In the 18th century, engineers had appreciated the fact that many materials could carry tensile loads, that they could deflect, and it had not seemed sufficient for them only to rely on stability for their gravity structures. They had learned the use of the beam and the use of timber and iron, so that there had been a logical switch from one field of study to another. For the subsequent 150 years the laws of elasticity had been expounded, but theories were no good if they could not be put into practice, and it had become evident that a more thorough understanding of the properties of the materials was necessary. During the 19th century particularly, very great attention had been paid to experiment. Engineers had learned more about the elastic properties of materials, fatigue, and so on, and consequently towards the end of the 19th or the beginning of the 20th century, engineers had inherited not only the 18th-century elastic theories, but also a vast fund of information on the properties of materials. In that way, the design of separate elements had been evolved.

At the present time, however, Dr Davey detected a marked change, in that a structure was no longer being viewed as a series of separate elements, but being specially designed and considered as a whole. To apply mathematics to those three-dimensional problems was exceedingly complex. It was so complex, in fact, that engineers were finding it necessary to design machines with which the mathematicians could work out the answers!

So far as the future was concerned, it would appear that not only did engineers have to deal with the structure as a whole, but they had also to consider questions of composite action, that was, the interaction of one material with another. In his view, during the next few decades the main concentration of work would be along those lines, mainly, of course, with a view to effecting the greatest economy possible so far as materials were concerned.

Mr R. P. Mears observed that the Author had stated in his conclusion that engineering design was an art, and had been practised as such—with no mean success—before any theory was developed. Dealing particularly

with arches, it appeared that Castigliano's theorem had been published in 1875, and in 1888 the three fundamental equations dealing with arches by elasticity had been produced. The lack of an elastic theory for dealing with fixed arches had been borne out in the very famous Presidential Address by Sir John Rennie in 1846.[82] He had stated : " A proper theory of the equilibrium of the arch, which shall satisfy all the conditions of the question, when applied to practice, may be said to be still wanting, though much valuable information may be derived from the scientific works of Hutton, Attwood, Moseley, Gwilt, and others, on the subject." He had spoken from a wealth of experience which few men in those days could possibly have equalled.

Mr Mears then showed a series of lantern slides, the first of which depicted the new London Bridge, which had been designed by Rennie the Elder and completed by his son, Sir John Rennie, in 1831. The arches of that bridge could not be bettered at the present time, even with the aid of elastic theory.

Another slide showed the old Southwark Bridge. The arches were of cast iron, and the centre span was 240 feet, with a rise of one-tenth. It had been begun in 1814 and finished in 1819. It had stood for 95 years and even then had only been taken down because some of its gradients had been too steep.

Mr Mears said that he had had the temerity to try to work out the stresses in the arches of the old Southwark Bridge. Stresses were reasonably low and appropriate, even under a live load of 100 lb. per square foot in just those positions where it would cause the greatest bending stresses. The l/b ratio was a little more than 40.

Referring to the arches of the old Waterloo Bridge,[83] he observed that it had been found during demolition, however the loading was increased or decreased as various materials were removed, a link polygon could always be drawn through the centroid of the arch stones.

The last slide (reproduced as *Fig. 8*) was a view of the famous Pont de Céret in south-east France, which had been built between 1321 and 1339. It had a span of 149 feet and the arch had a rise of 73 feet. At that time there had been no elastic theories or slide rules and other artificial aids. The appearance had been spoiled by the subsequent partial filling-up of the spandrels—which, according to Séjourné, was completely unnecessary.

Those illustrations would bear out the Author's assertion that engineering had been successfully practised as an art before any theory had been developed, and, in the hands of capable men, the lack of theory appeared to have been of very little account.

Dr H. Q. Golder said that he would go further than to support the Author's plea for a greater study of history by engineers, and Mr Gardner's

[82] Min. Proc. Instn Civ. Engrs, vol. V (1846), p. 28.

[83] E. J. Buckton and H. J. Fereday, " The Demolition of Waterloo Bridge." J. Instn Civ. Engrs, vol. 3, p. 472 (April 1936).

suggestion that students should have to pass an examination in it. He had held the theory for some years that the only subject which should be taught at school was history, for if it were done properly, all the other subjects would be taught at the same time.

Fig. 8

Pont de Céret

To give an example of what might happen, the question might be posed: "What happened in 1794 in engineering? State the background." The student would probably immediately recall that Woltmann had produced his trigonometrical equations for earth pressure, and might go on to sketch the background by saying that at that time Wordsworth and Beethoven were 24 years of age, and Mozart had died 3 years previously; that the Institution of Civil Engineers had not then been founded, although the Society of Civil Engineers, or the Smeatonian Society, had been in existence for more than 20 years. He would go on to mention that George III was on the Throne of England, where charming Georgian buildings were going up, and the Brothers Adam were in great demand; while in France the Terror was at its height.

Dr Golder said that during a recent short period of convalescence, he had managed to read most of the early works on earth pressure in the original, and most of them were to be found in the Library of the Institution. In the course of reading those early Papers he had discovered th

interesting fact that Woltmann had never mentioned Coulomb although Woltmann used exactly the same nomenclature. For instance, he and Coulomb had both used $\frac{1}{n}$ for the coefficient of friction, which seemed to be rather too much of a coincidence. Woltmann had probably known of Coulomb's Paper, but had not mentioned Coulomb, although he had mentioned various other writers. It would be interesting to learn from the Author exactly what had happened.

Dr Golder said that he would make his own guess, although he was aware that it was probably not true. In a previous Paper he had pointed out that Woltmann had been writing in Prussia, whose ruler—the weak son of Frederick the Great—had been carrying on a war with France, where the Terror was at its height under Robespierre; and that the work had been promoted by the Royal Scientific Academy of St Petersburg, whose patron, Catherine II of Russia, was engaged in a war with the French-inspired revolutionary government of Poland; there might therefore have been good political reasons why Woltmann had not mentioned the work of Coulomb!

It was important when a new subject started that someone should start writing its history at the time. That was much easier than going back 200 years later and finding out what had happened. Several new subjects related to engineering had started in the last 20 or 30 years. One was soil mechanics, and the Institution would be fortunate in having a lecture on that subject in the near future. Another subject, of course, was pre-stressed concrete. He suggested that many engineers who found that they had no mathematical ability to produce new theories and who did not have the opportunity of carrying out research, yet who desired to do something apart from their ordinary jobs, might choose a subject, study its history, and write it up. It was realized that, if it were a new subject, the writer might get involved with personalities, but it was not always necessary to publish the Paper having written it! In Dr Golder's view engineers could understand something better if they were told whence it came and how it arose.

Mr P. L. Capper observed that 2 or 3 years previously he had had to prepare a lecture on the history of the testing of materials, and he had found that researches to discover the old pioneers of the engineering world were very interesting.

There was a slight correction which he desired to make to the Author's statement on p. 388, referring to the chair of engineering at University College, London. That college, then known as London University, claimed to be the first in the field of engineering education in universities. The first list of appointments made in 1827 had included that of J. Millington as Professor of Engineering and the Application of Mechanical Philosophy to the Arts. Millington had resigned just before the opening of the college in October 1828 on the grounds of the Council's refusal to

guarantee his salary of £400 a year; but he had agreed to give occasional lectures. In 1833, the proposal to appoint a Professor of Engineering had been revived, but "provoked a protest from the Professor of Natural Philosophy (the Rev. W. McRitchie), who regarded it as an invasion of his province and a menace to his salary." [84] Special courses on civil engineering had been given by Ritchie and his successor Sylvester, supplemented by courses by the Professors of Mathematics and Chemistry. The Chair of Civil Engineering was eventually established in 1841 by the appointment of C. B. Vignoles.

Mr Capper supported the Author's remarks together with those of previous speakers in connexion with the use of graphical methods. They were very useful for many purposes in structural engineering work, and it was to be regretted that they seemed to have waned in popularity in academic circles. That might be because of the ever increasing curriculum of studies, which had tended to put draughtsmanship and drawing into rather secondary places.

On p. 377 of the Paper, the Author had mentioned the testing machine of van Musschenbroek. It was interesting that one of the tests which he had carried out had been to find the tensile strength of the horn of an ox. Mr Capper also quoted the description of Robert Hooke given in Chambers's Encyclopaedia, namely, "crooked in his person, he was upright in character, although solitary and penurious in his habits."

The Chairman said that he desired to raise one or two points which might have been touched upon in the discussion.

First of all, it was realized that the Author had had a big task in covering the whole field, but there was one group of problems which he did not seem to have touched at all, namely, those dealing with axially loaded beams. Those had been of great importance in the early days of aeroplane construction. It was difficult to say when the first of such problems had been solved, but the first English solutions seemed to be due to Dr Arthur Morley.

In the days of the biplane it had become necessary to extend the Theorem of Three Moments to take account not only of lateral loading, but of varying compressions in the different panels. That problem had been solved in Great Britain in 1915 by H. Booth and H. Bolas, and published as an Admiralty Air Department Paper. Mr Arthur Berry had later revised its form and had given a very satisfactory solution with tabulated functions which enabled it to be used more conveniently than in the original form. When the 1914–18 war ended and it had been possible to find out what had happened in other countries, it had been discovered that nearly all of them had produced independent solutions of the problem.

The Chairman mentioned that particular group of problems because th Author had pleaded for more graphical treatments, and it was doubtfu

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[84] H. Hale Bellot, "University College, London, 1826–1926." University c London Press, 1929.

if it was possible to find a more elegant graphical solution to a problem than the polar-diagram method of treating the axially loaded beam.

On p. 392 the Author had stated: "In all the methods of calculation which depend on strain energy, it is assumed that the material will deform in accordance with Hooke's Law." That was generally true, but the Author's attention was directed to the rather important exception. The first theorem of Castigliano stated that the differential coefficient of strain energy with respect to an external load was the displacement of that load in its line of action. That was true if load and displacement were connected linearly. A corresponding theorem that the differential coefficient of strain energy with respect to a displacement gave the load acting at the displaced point was, however, independent of the law connecting load and displacement.

The Chairman said that he was a little puzzled by Table 3, in which the Author listed the applications of strain energy. For example, the Theorem of Three Moments was ascribed to E. Clapeyron in 1857, but Clapeyron had not done it by strain energy but by comparison of slopes and deflexions. Similarly, Williot, to whom was ascribed the deflexion of a statically determinate frame in 1887, did not use strain energy; nor had strain energy been used by Beggs or Hardy Cross. It would be interesting to know why they had been classified in that way.

Mr R. J. Wilkins observed that the name of Robert Hooke had been mentioned several times in the discussion. Hooke's most famous publication was his Paper "*De Potentia Restitutiva*; or of Spring, explaining the Power of Springing Bodies," but he had also kept a diary, which was at present available,[85] and by reference to that diary it was possible considerably to amplify the human side of the discovery of his law.

Mr Wilkins then showed two slides giving quotations from Hooke's diary. The first passage read:

> "Thursday, Sept. 2nd, 1675.
> . . . To Tompions, told him my way of opposite springs which I had fully experimented before. All springs at liberty bending equall spaces by aequall increases of weight—borrowed Cox his tool of 50 foot and glasse. . . ."

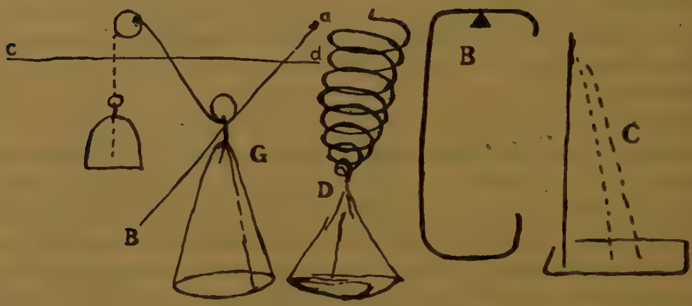
From other passages it appeared that the King had been very interested in Hooke's work, and there had been some suggestion that Hooke, together with the clockmaker Tompion, should construct a large chair suspended from a spring in order to weigh the King, but there was no evidence that it had actually been constructed.

Another passage read:

> "Wednesday, August 21st, 1678.
> To Dean Lloyd. Discorsed of Atlas, told all my way. Agreed

85 "The Diary of Robert Hooke, 1672–1680." Edited by H. W. Robinson and W. Adams. Taylor & Francis, London, 1935.

about showing Map to the King. To Sir Chr. Wren with him at Mans. Discoursd much about Demonstration of spring motion. He told me a pretty thought of his about a poysd weather glasse as at B. I told him an other as C. I told him my philosophicall spring scales as D. He told me of his mechanick rope scale as G, wherein the weights of the scale are found by the intersections of Cd by ab. . . ."



Mr Wilkins explained that Sir Christopher Wren had been Hooke's chief with whom he had had an agreement that they should exchange scientific notions. Mans was a coffee-house favoured by men of science. Hooke, incidentally, had been a very suspicious man; he had kept many things to himself and before delivering a Paper he would search the building thoroughly for spies!

In 1678 he had prepared his Paper on the theory of springs,[86] and on August 1st of that year there was an entry in the diary:

> " . . . I read my Theory of Springs and shewd the experiments to illustrate it, all were well pleasd. . . ."

It was interesting to note that Hooke's Law was so well expressed in his own words and that it had also been demonstrated as a matter of experimental fact.

The Author, in reply, endorsed Dr Skempton's tribute to the professional designers of mediaeval masonry buildings; those magnificent structures had not been just put together by country masons, but had been designed by men who had thoroughly studied the subject. The point which he (the Author) had tried to make in the introduction was not that those designers had been ignorant men, but that they had not been trained in what would now be understood as theory. Undoubtedly they had had experience, skill, a great deal of mechanical intuition, the courage to make daring experiments, and the wisdom to learn from their mistakes.

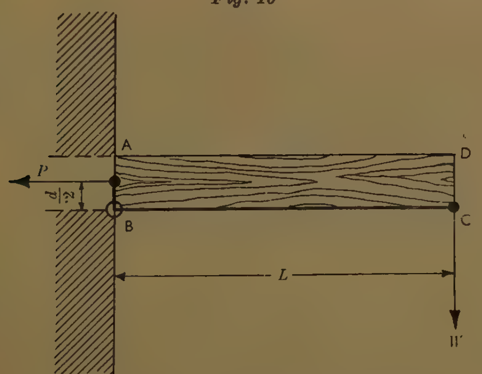
[86] See ref. 13, p. 399.

Mr Gardner held the view that all young engineers should be required to study, and to pass an examination in, engineering history. Unfortunately, it would be most difficult with already overloaded curricula to get such a proposal accepted at the present time. The Author's own suggestions to various educationists, to the effect that a move might be made in that direction, had not been favourably received. The most that could be hoped for was that people like Dr Skempton, Mr Capper, and others who lectured and who were interested in history, would make the most of their opportunities to introduce some history into their courses.

Mr Gardner had asked why it was that the philosophical approach of many investigators had been untenable, and in particular he had mentioned Galilei. No one who had read the discourses of Galilei could in any way accuse him of lacking the philosophical knowledge of the time. His works were in the form of discourses between a follower of Aristotle (who at the end of each chapter always got the worst of the argument), a neutral observer, and another who advanced Galilei's own views. That was a not uncommon way at that period of publicizing new theories. When considering Galilei's treatment of the cantilever it should not be forgotten that he had been breaking entirely new ground on which traditional philosophy had thrown no light whatever.

At failure, the beam A B C D (*Fig. 10*) tore across at the section A B and rotated about the edge B which apparently served as the fulcrum of a virtual bell-crank lever. Since the whole section A B tore apart, it had

Fig. 10



GALILEI'S THEORY OF THE BEAM

been natural to assume that the whole section was in tension with the resultant resisting force acting through the mid-point of the section. It would immediately occur to a student of the present day, accustomed to thinking in terms of stress-distribution, that that implied an extremely

high compressive stress at B, but in Galilei's time no one had yet thought about stress-distribution. Not until about 40 years later had both Hooke and Mariotte noted that the distortion of the resisting fibres varied continuously across the section, and even Mariotte had seen nothing illogical in supposing both that tensile forces would vanish and that maximum compressive forces would develop concurrently at B. In historical studies the achievements of the pioneers had to be assessed against the background of their own time. The extent of present knowledge was founded only upon the accumulation of the contributions of previous centuries.

The experimental curves in Mr Gardner's graph (*Fig.* 7) for axially-loaded columns with a high slenderness ratio were of nearly the same form as the computed curves of *Fig. 3*. They supported the view that, whatever the physical defect whereby any particular column fell short of Euler's ideal assumptions, under test it sooner or later deflected as though the load had been slightly eccentric.

Mr Bondy had drawn attention to the alternative ascriptions of the same theory in different countries. There was always a tendency for people to name a theory or characteristic after the expositor they knew best. Although Cremona's book—alone among the major continental works on graphic statics—had been translated into English, the introduction of this subject was ascribed (and probably correctly) to Clerk Maxwell. It was true that the constant known as "Young's Modulus" had been used by Euler in one of his St Petersburg papers (1778), but it was only after Dr Thomas Young's work (1807) that its importance had been recognized. Young's own modulus had been actually the *weight* of a column of the material which would evoke a hypothetical unit strain in a specimen of unit cross-section. Both priority and form would justify the term "Euler's Modulus," but usage had decreed otherwise. Poncelet's graphic determination of earth pressures had been earlier than Rebhann's. Both depended upon the same theory, but Rebhann's diagram was easier to construct and was still widely used in Britain and abroad.

Mr Mears had quoted from Sir John Rennie's Presidential Address of 1846 a reference to the work of various English writers on the stability of arches. Sir John's selection of authorities was curious; Attwood and Hutton, neither of them engineers, had merely elaborated La Hire's Smooth Voussoir Theory, long out of date in France. The Rev. Henry Moseley's treatment had been sounder but highly mathematical and laborious. Moseley had indeed found, after completing his own work, that Coulomb had arrived at the same solution by simpler means about 70 years previously! [87] The Author had outlined elsewhere with references the subsequent graphical treatment of the arch in Britain up to about 1880.[88]

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[87] H. Moseley, "On The Theory of the Arch" (p. 23), in "The Theory, Practice and Architecture of Bridges," vol. I, Weale, London, 1843.

[88] S. B. Hamilton, "Pont-y-ty-Prydd: Notes on the Technical Significance of a Remarkable Bridge," Trans. Newcomen Soc., vol. XXIV (1943–45), p. 131.

Dr Golder had asked why Woltmann had not mentioned Coulomb. The Author had not studied Woltmann sufficiently closely to discover why he had mentioned some men and not others but suspected that Dr Golder himself had provided the correct answer.

Mr Capper had pointed out that the Paper did an unintentional injustice to University College, London, in the matter of priority. The Author had given in the Paper the dates of foundation of fully-organized courses of instruction in the two London colleges; some attention had previously been paid to mechanical science in both places. King's College might similarly have objected to reference to Rankine's text-books without mention of the prior publication of two notable contributions to technical literature by members of their staff: a composite work in three stout quarto volumes on "The Theory, Practice and Architecture of Bridges,"[89] and Moseley's "Mechanical Principles of Engineering and Architecture."[90] Both works had appeared in 1843. The paragraph in which the reference appeared had been intended, however, to call attention to the interest which was being taken at that time in the theoretical training of engineers; it had not been intended to deal comprehensively with the means adopted to that end. Otherwise it should have mentioned the work of Dr George Birkbeck (1776–1841) from 1799 onwards at Anderson's College, Glasgow; the foundation in 1824 of the Birkbeck Institute in London, and later of other mechanics institutes which had developed into famous technical colleges elsewhere. The difficulty in preparing the Paper throughout had been to decide how much could reasonably be omitted; and the Author had been gratified to find that, on the whole, his critics in the discussion had appeared to be fairly satisfied with his selection.

The Chairman had supplied information on a group of problems to which the Author had not done full justice in the Paper, and for that he thanked him. In Table 3, a number of people had been quoted whose work had seemed to be in the line of development traced in the Table although some of them (as the Chairman had correctly remarked) had not employed strain-energy methods. The Author's excuse was that he had not been able to think of a better short heading for the Table.

Correspondence on this Paper is closed. No contributions may now be accepted.

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[89] See ref. 87, p. 418.

[90] H. Moseley, "The Mechanical Principles of Engineering and Architecture." Longmans, London, 1843.

Paper No. 5887

"Experimental Analysis of Space Structures, with Particular Reference to Braced Domes; with a Note on Stresses in Supporting Ring-Girders"

by

Zygmunt Stanislaw Makowski, Dip. Ing., and Professor Alfred John Sutton Pippard, M.B.E., D.Sc., M.I.C.E.

(Ordered by the Council to be published with written discussion) †

SYNOPSIS

Indirect methods of experimental stress analysis based on Clerk Maxwell's reciprocal theorem are often used for plane frames but seldom for space structures. The Dome of Discovery and other structures of similar type are used in this Paper to demonstrate the general applicability of experimental analysis. A simple braced dome was analysed theoretically and the results are compared with those determined from a model made of $\frac{1}{8}$-inch steel wire. Displacements of known amounts in three mutually perpendicular directions were imposed on the foot of one of the supporting struts and the resulting nodal displacements were measured by cathetometers. The ratios of these to the imposed displacements gave influence coefficients of reactions. The results agreed well with the calculated values and the method was then applied to a model of the Dome of Discovery. Certain members of this model were fitted with small turnbuckles so that they could be altered in length by known amounts. The ratios of the component displacements produced at any node to the alteration in bar length gave influence coefficients of stress in that member. The results again agreed well with those calculated by relaxation methods.

Formulae are given for the resultant actions in the ring-girder and these were satisfactorily checked experimentally by comparing measured and calculated displacements at points around a model ring.

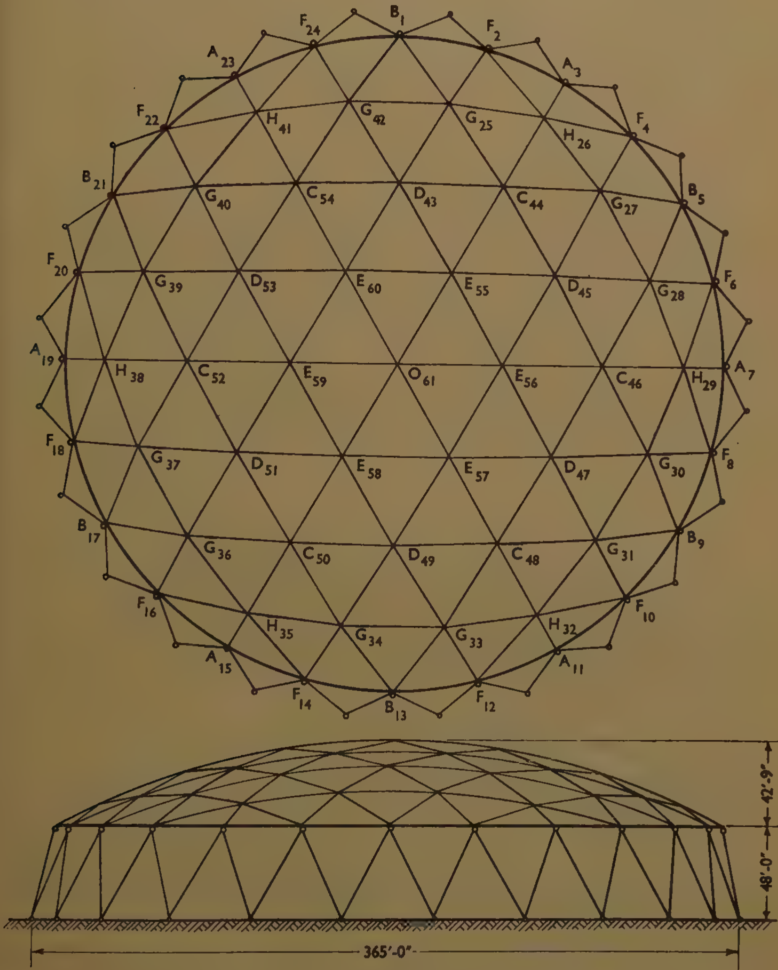
The stress analysis of plane or space frames is straightforward provided that they are statically determinate, but when they are redundant, computation may be lengthy and complicated and in such circumstances experimental methods of analysis are often of great value. The indirect method, using Clerk Maxwell's reciprocal theorem, is well-known in its application to plane frames,[1] but it has not commonly been used for space structures. The method is, however, equally applicable to space frameworks and this Paper gives details of the technique used and the results obtained for certain particular structures.

† Correspondence on this Paper should be received at the Institution by 1st April 1953, and will be published in Part III of the Proceedings. Contributions should be limited to about 1,200 words.—Sec. I.C.E.

[1] The references are given on p. 440.

The Dome of Discovery, built for the Festival of Britain, 1951, attracted attention to a type of engineering structure which was new in Great Britain, and this type of structure well illustrates the value of experimental

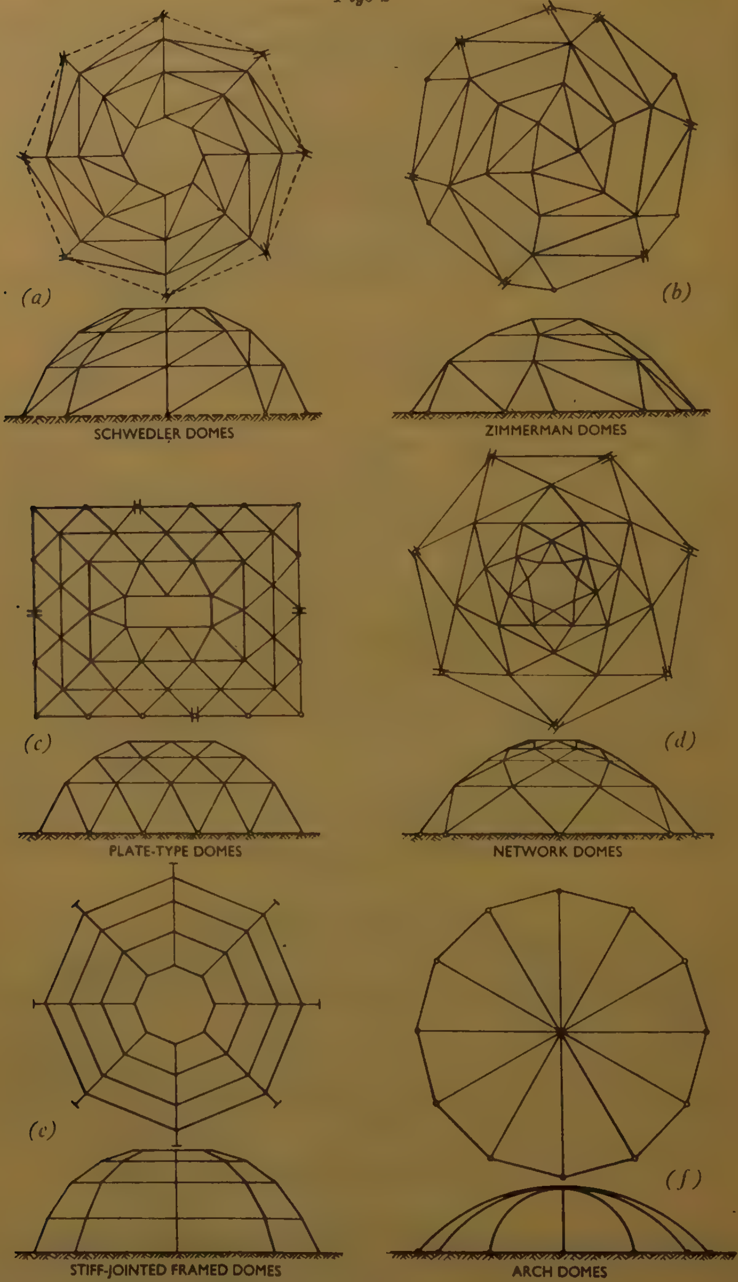
Figs 1



THE DOME OF DISCOVERY

methods of analysis. It should not, however, be concluded that such methods are only applicable to braced domes; they can be used for many other space structures. To complete the Paper an analytical treatment,

Figs 2



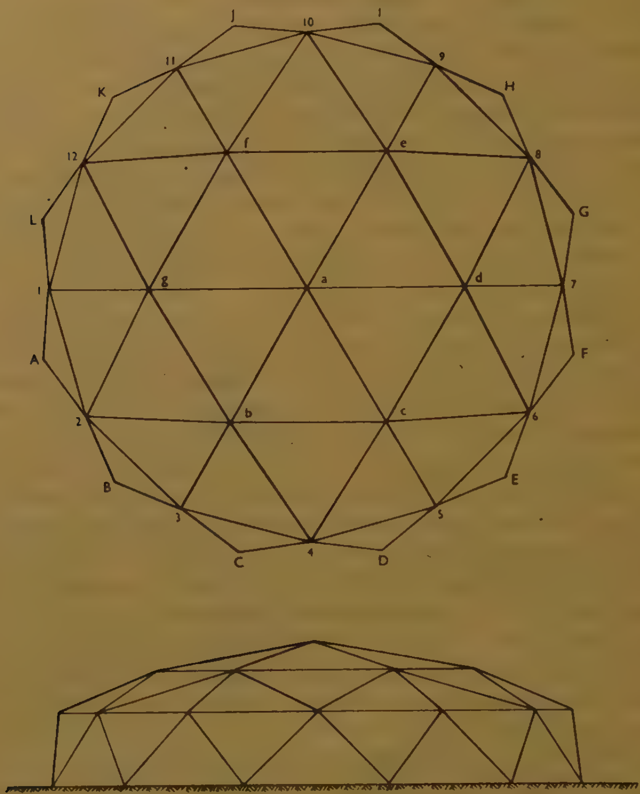
TYPES OF BRACED DOMES

by an approximate method, is included for the supporting ring-girder which often forms a part of such structures. The Dome of Discovery consisted of a number of intersecting circular ribs springing from a circular ring-girder which was supported on a number of bipods hinged at the feet, as shown in *Figs 1.* Alterations in the diameter of the ring-girder caused by temperature changes thus produce no stresses in the bipods, and, if the bracing members of the dome and the ring-girder are of the same material, the structure will not be stressed by temperature changes. Actually the ribs and the ring-girder were of different materials, so that temperature stresses were induced, but once the principles of the method to be described have been understood there should be no difficulty in calculating them.

The structure of a braced dome is usually composed of members lying on the surface of a solid of revolution or of straight members connecting points lying on that surface. Typical designs of earlier structures are shown in *Figs 2*, and methods for calculating the stresses in them have been described elsewhere.[2] Most of the examples shown are spherical in form, but they might equally well be paraboloidal or of other suitable shape. In the Dome of Discovery, the biggest and most recent of such structures, the bracing members considered individually were arched ribs, and if they had acted independently, stress analysis would have presented no special problems; they were, however, connected together at all points of intersection, and in consequence the structure exhibited a very high degree of redundancy. Analysis by classical methods such as strain energy, slope-deflexion, and so on, leads to such a number of simultaneous equations that solution is impracticable. Relaxation treatment is possible, but the computation is considerable and requires workers skilled in the particular technique; this method was in fact applied to the Dome of Discovery.[3] Approximate methods of analysis can be devised for most engineering problems, but the danger of over-estimating the stresses can only be avoided at the risk of under-estimating them and so losing the economical advantage which is one of the main features of the design. In view of these difficulties it appeared to the Authors that it was worth while investigating the possibilities of experimental stress determination. Two different methods may be adopted. An exact scale model of a preliminary design can be loaded and the stresses in the various members measured by means of suitable extensometers; this, whilst practicable, is expensive in labour and material and also in the time required to make and reduce the strain measurements. A second and much simpler approach is the indirect one based on Clerk Maxwell's reciprocal theorem. This is a well-known practical experimental technique for plane structures such as arch ribs, portals, and building frames,[1] but its application to space frames has not so far been much exploited.

Figs 3 represents a space frame supported at points A, B, C, etc., while a, b, c, etc., are nodes of the structure.

Figs 3



SIMPLE DOME

If the support at any point, say A, is released and an arbitrary dis-placement Δ is imposed in the X direction so that it has no component displacements in the directions of the other mutually perpendicular axe Y and Z, then, if any node, such as a, is thereby displaced δ in some specified direction, it follows from Clerk Maxwell's theorem that a load W applied in that direction at a will produce a reaction $-\dfrac{W\delta}{\Delta}$ in the X direc-tion at A.

If, therefore, an elastically similar flexible model of the structure i made and if displacements are successively applied in three mutuall perpendicular directions to a released support point, the resulting dis placements of the nodes can be measured and influence coefficients c reactions can be calculated, from which, in simple structures, all the stresse

can be determined. In more complex structures this will not be possible and loads in certain individual members must also be found experimentally. The technique will be described in detail by reference to actual cases but it is first necessary to explain how elastic similarity may be obtained.

If the bracing is an intersecting lattice, the main stresses in its members will be direct tensions or compressions; the stresses in the ring-girder on the other hand will be mainly due to bending. Hence the effects of energy arising from bending of the bars and from direct stresses in the ring may be neglected.

For any load system on the prototype structure, let P be the force in any bar of the lattice and M the bending moment at any point in the ring-girder. The component deflexion of a node p in the direction of W, the load acting there, is then:

$$\delta = \sum \frac{PL}{AE} \cdot \frac{\partial P}{\partial W} + \int \frac{M}{EI} \cdot \frac{\partial M}{\partial W} ds$$

where AE is the extensional rigidity of the bar carrying P, and EI is the flexural rigidity of the ring-girder at the point considered. The summation extends to all members of the lattice and the integration to the whole of the ring-girder. Suppose now that a model is made of the prototype such that the extensional rigidity of a lattice bar is $k_1 AE$ and the flexural rigidity of the ring-girder is $k_2 EI$, whilst the linear scale of the model is k_3 times and the load scale k_4 times that of the prototype, so that the load at p is $k_4 W$.

Then the deflexion of p on the model is:

$$\delta' = \frac{k_3 k_4}{k_1} \sum \frac{PL}{AE} \cdot \frac{\partial P}{\partial W} + \frac{k_3{}^3 k_4}{k_2} \int \frac{M}{EI} \cdot \frac{\partial M}{\partial W} ds.$$

If the displacements of the prototype and those of the model are to be proportional, it is essential that:

$$\frac{k_3 k_4}{k_1} = \frac{k_3{}^3 k_4}{k_2}$$

which becomes

$$\frac{k_3{}^2 k_1}{k_2} = 1 \quad \ldots\ldots\ldots\ldots \quad (1)$$

so that

$$\frac{\delta}{\delta'} = \frac{k_3 k_4}{k_1}$$

and the model and prototype will then be elastically similar.

It will be observed that the assumption that the energy in the lattice arises from direct stress only, and that in the ring-girder from bending stress only, removes all restrictions as to the actual cross-sectional shapes of the members; provided that the values of AE and EI satisfy the scale requirements of equation (1) their actual cross-sections do not matter and simple models can be constructed.

The first example chosen is shown in *Figs 3*. It consists of a comparatively simple structure which is amenable to analytical solution and was chosen to obtain a comparison between calculated results and those found by the experimental method. There was no prototype and the stress distribution in the actual structure was calculated for two particular load systems.

The structure was made from ⅛-inch-diameter steel rod and all joints were welded. The total height was 1 foot 3 inches and the span 5 feet 3 inches. The feet of the supporting struts were fitted with ball-joints, any one of which could be mounted in the space deformeter shown in *Fig. 4*.* The general view of the model with the space deformeter fixed at one of the supports is shown in *Fig. 5*. The joint thus mounted could be displaced in turn in each of three mutually perpendicular directions by known amounts Δ_x, Δ_y, and Δ_z and the corresponding component displacements of any node of the structure, δ_x, δ_y, and δ_z, could be measured; x, y, and z denote arbitrarily chosen co-ordinate directions, z being vertical. Then, if a unit load acts vertically at any joint the three component reactions at the support are $-\frac{\delta_z}{\Delta_x}$, $-\frac{\delta_z}{\Delta_y}$, and $-\frac{\delta_z}{\Delta_z}$, and if unit loads act parallel to the x- and y-axes the component reactions are respectively $-\frac{\delta_x}{\Delta_x}$, $-\frac{\delta_x}{\Delta_y}$, $-\frac{\delta_x}{\Delta_z}$ and $-\frac{\delta_y}{\Delta_x}$, $-\frac{\delta_y}{\Delta_y}$, $-\frac{\delta_y}{\Delta_z}$. These influence coefficients are conveniently denoted by symbols such as C_{pq} where p and q denote the directions in which δ and Δ respectively are measured; thus C_{xx} is $-\frac{\delta_x}{\Delta_x}$, C_{yz} is $-\frac{\delta_y}{\Delta_z}$, etc.

It will be observed that in the particular model it is unnecessary, except as a check, to displace more than one joint since the structure is completely symmetrical.

After tables of influence coefficients have been compiled, the reactions at all supports can be found for any load system on the dome. If for example W_a, W_b, etc., act vertically at joints a, b, etc., the vertical reaction at A is:

$$Z_A = W_a(C_{zz})_a + W_b(C_{zz})_b + \ldots + W_q(C_{zz})_q + \ldots + W_n(C_{zz})_n$$

where $(C_{zz})_q$ represents the influence coefficient for A under a vertical load at q.

Similarly the remaining component reactions at A are:

$$X_A = W_a(C_{zx})_a + W_b(C_{zx})_b + \ldots + \text{etc.}$$

and

$$Y_A = W_a(C_{zy})_a + W_b(C_{zy})_b + \ldots + \text{etc.}$$

If the reactions arising from horizontal loads are required, similar Tables must be compiled for C_{xx}, C_{xy}, C_{xz}, and C_{yx}, C_{yy}, C_{yz}; the procedure for determining X_A, Y_A, and Z_A is then the same.

* *Figs 4* to *7* inclusive are all photographs, and appear between pp. 436 and 437.

When displacements were imposed on the model, some of the compression members tended to buckle, with a consequent reduction in their effective modulus of elasticity. This was prevented by attaching splints made of lengths of $\frac{1}{8}$-inch-diameter rod to those members by soft wire; since the splints were unconnected to the joints, they carried no axial load but stabilized the struts against buckling.

To determine the force in a bar of the loaded structure, the distance between the nodes connected by that bar must be altered by a known amount Δ'. This was done by inserting very small turnbuckles in those bars which were to be analysed.

A number of those turnbuckles in different bars can be seen in *Fig. 5*. The displacements of the various joints of the structure caused by the change of length of the bar were measured and the forces in the bar were then $-\left(\frac{\delta_x}{\Delta'}\right)_q$, $-\left(\frac{\delta_y}{\Delta'}\right)_q$, and $-\left(\frac{\delta_z}{\Delta'}\right)_q$ for unit loads applied at joint q in the directions x, y, and z respectively. The corresponding influence coefficients of stress in a bar, cd for example, are denoted by $(C_x')_{cd}$, $(C_y')_{cd}$, and $(C_z')_{cd}$. Displacements of the joints were quickly obtained by means of cathetometers.

Two methods of approach have been outlined:

(*a*) displacement of supports and calculation of reaction coefficients; and

(*b*) direct determination of the forces in particular members by alteration of lengths in individual bars.

The first method is specially useful in the case of simple space structures on a limited number of supports. In these, a knowledge of the component reactions eliminates the redundancy and usually leads to the stresses in all members being computed very simply.

In space structures having a large number of supports, the stresses in the supporting struts caused by unit loads acting at various joints of the structure are very small and even a small error in the determination of a strut load can introduce quite a large error in the forces in members at a distance from the strut.

In structures with very stiff ring-girders or a complicated and highly redundant lattice, it is of only limited help to know the magnitudes of the forces in the supporting struts; the direct determination of loads in the members, on the other hand, gives results simply and quickly. This method can be used for any structure with straight members and is especially valuable for highly redundant structures. The load in a member determined by this method does not influence the accuracy of the result for loads in neighbouring bars, since every member is treated independently.

In both methods large displacements are recommended, since cathetometers, micrometer screws, or even ordinary dial gauges are accurate enough for measuring joint displacements, which in the cases to be described were of the order of $\frac{1}{2}$ inch.

If, however, displacements are too large, errors may arise since the geometry of the structure may be appreciably altered. It is specially important to avoid this in models of flat domes.

It was found by experience that the best results were obtained if the alterations in the length of a member, Δ', were made in a number of steps rather than in a single one. Alterations in the configuration of the framework caused by the imposition of large displacements were allowed for by increasing and decreasing the lengths of the bars by equal increments about their unstrained initial values. When these alterations in length were plotted against the corresponding mean values of δ_x, δ_y, and δ_z, good straight lines resulted whose slopes gave the required influence coefficients. Deviation from a straight line indicated that buckling had occurred in one or other of the compression bars in the frame.

The structure shown in *Fig. 5* was analysed theoretically by a relaxation process on the assumption that bending of the lattice bars was negligible, and Table 1 shows a comparison between calculated stresses and those obtained experimentally under vertical point loads at the centre of the dome and at joint d.

The agreement was good and showed that the experimental approach is sound.

It was next decided to construct a simple model of the Dome of Discovery and to investigate the possibility of determining the stresses in it experimentally.

On the assumptions that the ribs and the supporting struts would take axial loads only and that the heavy steel ring-girder would be subjected to bending moments only, a geometrically and elastically similar model was made. The ribs and the ring-girder were made from mild-steel rods and the bipods from thin brass tubes.

The supporting struts were pin-connected to the ring-girder and to the support bases by means of small flexible pins, enabling small adjustments to be made to the lengths of struts and heights of joints. The support plates were so constructed as to fix the feet of the bipods in position, but at the same time to ensure their freedom of rotation, without the possibility of play between the ball-joint and base plate.

The span of the model was 10 feet 6 inches and a general view is shown in *Fig. 6*.

The model was made in two separate parts :

(*a*) the upper part, consisting of the dome bracing ; and
(*b*) the lower part, consisting of the heavy ring-girder connected to its supporting bipods.

In the first experiment the upper part without the ring-girder was fixed to rigid supports and the loads in all bars induced by a unit load at the centre of the dome were found. A view of the model in that state is shown in *Fig. 7*.

TABLE 1.—COMPARISON OF ANALYTICAL AND EXPERIMENTAL STRESSES CAUSED BY 10,000 LB. LOAD

(*a*) *Load at* d

Bar	Analytical : lb.	Experimental : lb.
$a-d$	− 9,340	− 10,750
$a-e$	+ 6,676	+ 6,270
$a-f$	− 1,290	− 1,050
$a-g$	− 1,355	− 1,480
$d-e$	− 10,952	− 10,800
$e-f$	+ 457	+ 500
$f-g$	+ 869	+ 836
$d-7$	− 7,914	− 9,410
$e-9$	− 1,993	− 1,965
$f-11$	− 696	− 635
$g-1$	− 238	− 400
$d-8$	− 15,243	− 15,700
$e-8$	+ 9,497	+ 8,900
$e-10$	− 1,919	− 1,920
$f-10$	+ 313	+ 640
$f-12$	− 85	− 450
$g-12$	− 270	− 330
$1-12$	+ 357	+ 740
$12-11$	+ 707	+ 1,160
$11-10$	+ 1,542	+ 1,476
$10-9$	+ 3,260	+ 3,460
$9-8$	+ 3,397	+ 3,020
$8-7$	+ 13,597	+ 14,800
$L-12$	+ 245	+ 200
$10-J$	− 608	− 590

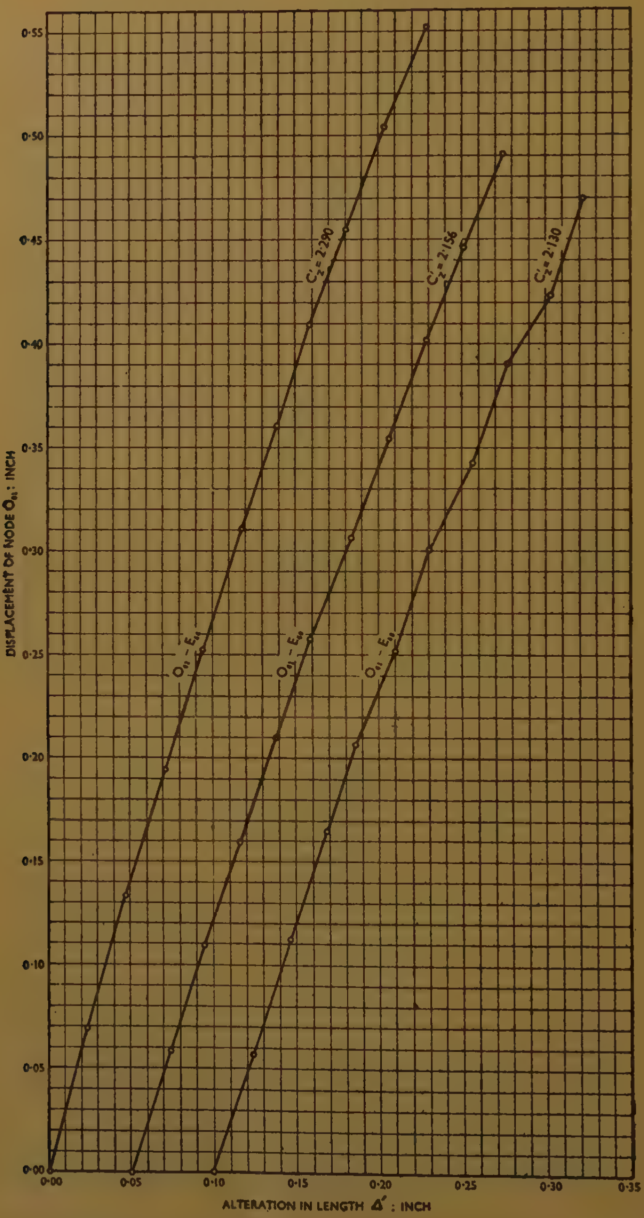
(*b*) *Load at* a

Bar	Analytical : lb.	Experimental : lb.
$f-12$	− 1,403	− 1,373
$a-c$	− 8,434	− 8,500
$c-d$	+ 5,142	+ 4,370
$5-4$	+ 3,815	+ 3,570
$d-7$	− 2,178	− 2,360
$e-9$	− 2,273	− 2,360
$11-10$	+ 3,815	+ 3,790
$b-g$	+ 5,142	+ 5,400
$a-g$	− 8,434	− 8,970

The lattice dome was then connected with the ring-girder, and the stresses in the same bars induced by a unit load at the same point were again determined experimentally. They were found to be very much as in the first experiment, indicating that the ring-girder could be assumed to be approximately rigid.

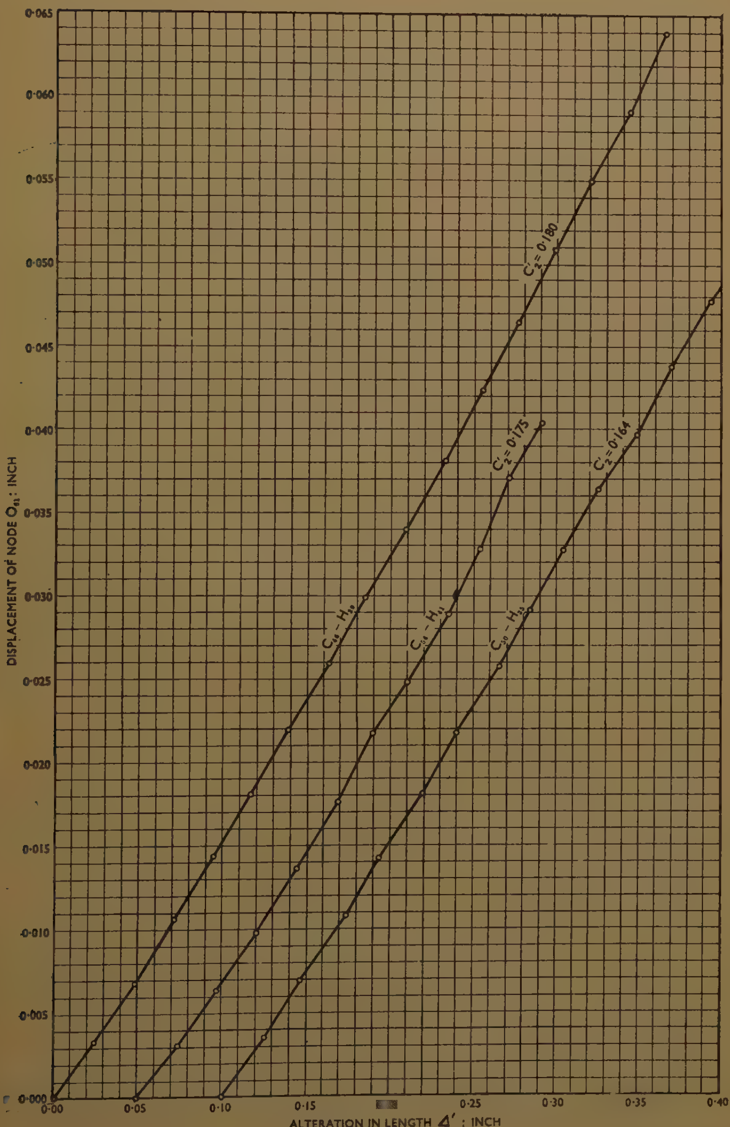
The structure was analysed for the case of a unit load acting vertically at point O_{61} and *Figs* 8 give three sets of experimental results for the

Fig. 8 (a)



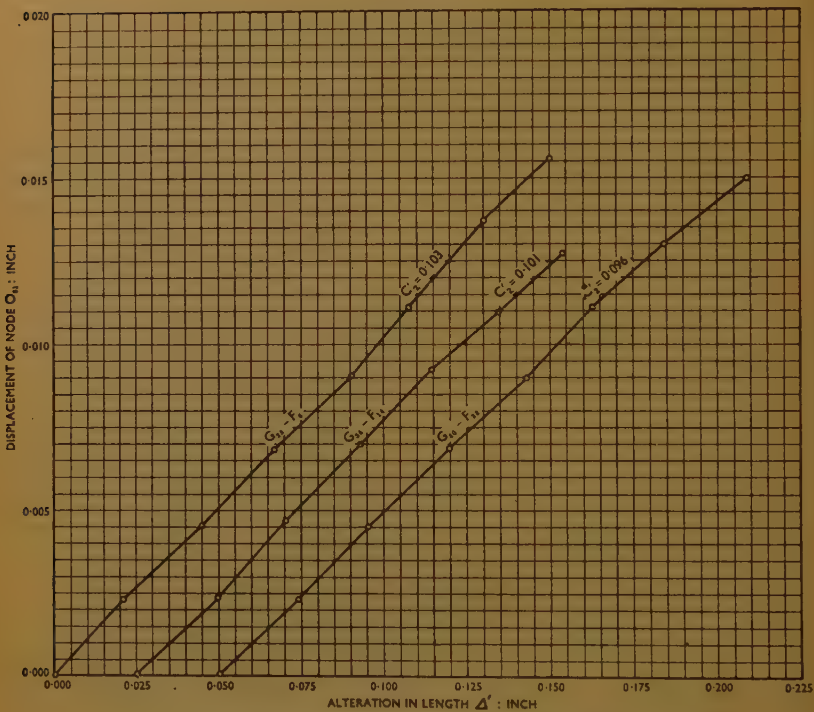
DETERMINATION OF INFLUENCE COEFFICIENTS

Fig. 8 (b)



DETERMINATION OF INFLUENCE COEFFICIENTS

Fig. 8 (c)



DETERMINATION OF INFLUENCE COEFFICIENTS

determination of the loads in members O–E, C–H, and G–F respectively. These curves give the vertical displacements of O_{61} for a range of alterations in length of the bars considered. They are shown as examples of the results obtained ; similar curves were found for all other bars.

Taking the straight part of the curve in each case, the value of the load in the bar under unit load at O_{61} was determined as previously explained. It is of interest to know that O–E is one of the most heavily stressed bars under the action of a load at O_{61}, while the stress in G–F is only about 5 per cent of that in O–E. The lines obtained are excellent and inspire confidence in the result.

Table 2 gives the experimental loads in members of the dome under the unit vertical load at O_{61}, and for comparison with these a complete relaxation analysis was made. The results of this analysis are given in the same Table and the general agreement between experiment and calculation is excellent. In some cases apparently large discrepancies appear, but it is probable that they arise from certain irregularities in the erection of the

TABLE 2.—LOADS IN BARS UNDER UNIT VERTICAL LOAD AT O_{61}

Bars	Experimental	Mean	Analytical
$O - E_{58}$	− 2·130		
$O - E_{60}$	− 2·156	− 2·192	− 2·184
$O - E_{56}$	− 2·290		
$E_{58} - E_{59}$	+ 1·545		
$E_{56} - E_{57}$	+ 1·515	+ 1·527	+ 1·514
$E_{60} - E_{55}$	+ 1·520		
$E_{60} - D_{53}$	− 0·143		
$E_{56} - D_{45}$	− 0·204	− 0·137	− 0·139
$E_{58} - D_{49}$	− 0·065		
$E_{57} - C_{48}$	− 0·633		
$E_{55} - C_{44}$	− 0·526	− 0·571	− 0·543
$E_{59} - C_{52}$	− 0·555		
$C_{50} - D_{51}$	+ 0·307		
$C_{54} - D_{43}$	+ 0·310	+ 0·327	+ 0·308
$C_{46} - D_{47}$	+ 0·366		
$C_{50} - H_{35}$	− 0·164		
$C_{46} - H_{29}$	− 0·180	− 0·173	− 0·250
$C_{54} - H_{41}$	− 0·175		
$C_{48} - G_{33}$	0		
$C_{44} - G_{27}$	− 0·034	− 0·003	~ 0
$C_{52} - G_{39}$	+ 0·026		
$D_{47} - G_{31}$	− 0·139		
$D_{51} - G_{37}$	− 0·158	− 0·157	− 0·137
$D_{43} - G_{25}$	− 0·175		
$G_{34} - G_{33}$	+ 0·170		
$G_{40} - G_{39}$	+ 0·190	+ 0·183	+ 0·172
$G_{27} - G_{28}$	+ 0·189		
$H_{32} - G_{31}$	+ 0·133		
$H_{38} - G_{37}$	+ 0·108	+ 0·104	+ 0·135
$H_{26} - G_{25}$	+ 0·070		
$H_{32} - A_{11}$	− 0·121		
$H_{38} - A_{19}$	− 0·088	− 0·099	− 0·156
$H_{26} - A_{3}$	− 0·087		
$H_{29} - F_{8}$	0		
$H_{35} - F_{16}$	+ 0·018	+ 0·019	+ 0·012
$H_{41} - F_{24}$	+ 0·039		
$G_{28} - F_{6}$	− 0·103		
$G_{34} - F_{14}$	− 0·101	− 0·100	− 0·094
$G_{40} - F_{22}$	− 0·096		
$G_{31} - B_{9}$	− 0·060		
$G_{37} - B_{11}$	− 0·039	− 0·059	− 0·031
$G_{25} - B_{1}$	− 0·078		

model. The Dome of Discovery was remarkably flat, as will be evident from a comparison with the domes shown in *Figs 2*, and the effect is to make it very sensitive to small errors in erection.

Six bars meet at any point, and since this apex is only slightly raised above the plane through the other extremities of the bars some toggle action may develop in which displacements and the forces required to cause them no longer follow a linear law. The necessity for care in the construction and erection of a model of this type is therefore obvious. It should be remembered that the relaxation approach to the problem will be correspondingly delicate and a large number of relaxations would probably be necessary to obtain a satisfactory balance.

As already mentioned, the loads in the bars of the dome when the structure was complete differed very little from those when the dome itself was attached to a rigid base without the ring-girder, and so for a ring-girder as stiff as the one under consideration it would be legitimate to treat it as an approximately rigid support. Where such an assumption is justified the analytical computations are greatly simplified, since the work can be divided into separate analyses of the dome and the ring-girder with its supports.

Ring-Girder : Forces in Plane of Ring

The ring-girder of the Dome of Discovery was carried on the hinged bipods of the supporting structure which ensured that while no restraint was imposed upon radial displacements, any tangential movements were resisted by the triangulated structure provided by the bipods. A solution of the essential problem was originally developed for a special form of aeroplane gun-mounting and this served for the development of formulae applicable to the present case.[4] The essential assumption is that the ring supports are sufficiently numerous to be replaced by a continuous medium offering no resistance to radial displacements but exerting a uniform resistance to tangential movements.

Suppose a ring-girder of radius R and flexural rigidity against bending in its plane EI to be supported on N bipods each of which is hinged at the base to allow free radial displacements of its apex.

Let W be the force which, applied to the apex of one bipod in a direction tangential to the ring-girder, produces unit tangential displacement. The N discrete supports are assumed to be replaced by a continuous restraining medium having a uniform intensity of resistance $\frac{NW}{2\pi R}$.

$$\text{Let } q = \frac{NR^3W}{2\pi EI}$$

$$Q_1 = \frac{1}{27} + \frac{q}{2}$$

$$Q_2 = \left(\frac{q^2}{4} + \frac{q}{27}\right)^{\frac{1}{2}}$$

$$X = (Q_1 + Q_2)^{\frac{1}{3}} + (Q_1 - Q_2)^{\frac{1}{3}}$$

and $$\alpha = \tfrac{1}{2}(3X + 2)^{\frac{1}{2}}\,;\ \beta = \left(\frac{3X-2}{12}\right)^{\frac{1}{2}};\ \gamma = 2\beta.$$

If ψ is the angular distance of any point on the ring from an arbitrary datum and a load acts in the plane of the ring at $\psi = \pi$, the outward radial displacement u and the anticlockwise displacement v at the point defined by ψ are given by the following equations :

If P_R acts radially inward,

$$u = J \cosh \gamma\psi + C \cos \alpha\psi \cosh \beta\psi + D \sin \alpha\psi \sinh \beta\psi \quad . \quad . \quad (2)$$

and $$v = -\left[\frac{J}{\gamma}\sinh \gamma\psi + \frac{C\beta - D\alpha}{\alpha^2 + \beta^2}\cos \alpha\psi \sinh \beta\psi + \frac{C\alpha + D\beta}{\alpha^2 + \beta^2}\sin \alpha\psi \cosh \beta\psi\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where $$J = \frac{P_R R^3}{2EI} \cdot \frac{\gamma \operatorname{cosech} \gamma\pi}{(\gamma^2 + 1)(3\gamma^2 + 1)};$$

$$C = -J\Psi_1\,;\ \text{and } D = J\Psi_2$$

The bending moment at any point in the ring is :

$$M = \frac{EI}{R^2}\left[J(\alpha^2 + \beta^2)\cosh \gamma\psi - 2\beta\{(\beta C - \alpha D)\cos \alpha\psi \cosh \beta\psi + (\beta D + \alpha C)\sin \alpha\psi \sinh \beta\psi\}\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

and the intensity of tangential force exerted on the ring is :

$$p = \frac{NWv}{2\pi R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

If P_T acts tangentially clockwise,

$$u = B \sinh \gamma\psi + F \cos \alpha\psi \sinh \beta\psi + S \sin \alpha\psi \cosh \beta\psi \quad . \quad . \quad (6)$$

and $$v = -\left[\frac{B}{\gamma}\cosh \gamma\psi + \frac{F\beta - S\alpha}{\alpha^2 + \beta^2}\cos \alpha\psi \cosh \beta\psi + \frac{F\alpha + S\beta}{\alpha^2 + \beta^2}\sin \alpha\psi \sinh \beta\psi\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

where $$B = \frac{\pi P_T}{NW} \cdot \frac{\gamma^2(\gamma^2 + 1)\operatorname{cosech} \gamma\pi}{3\gamma^2 + 1};\ F = -B\Psi_4;\ S = B\Psi_3;$$

and $$M = \frac{2P_T R}{q} \cdot \frac{\beta^2(\alpha^2 + \beta^2)^2}{(4\alpha^2 - 3)}\left[\frac{\sinh \gamma\psi}{\sinh \gamma\pi}\right.$$

$$-\frac{2}{\alpha(\cosh 2\beta\pi - \cos 2\alpha\pi)}\Big\{(\alpha\cos\alpha\pi\sinh\beta\pi + 3\beta\sin\alpha\pi\cosh\beta\pi)\cos\alpha\psi\sin\beta\psi$$

$$-(3\beta\cos\alpha\pi\sinh\beta\pi - \alpha\sin\alpha\pi\cosh\beta\pi)\sin\alpha\psi\cosh\beta\psi\Big\}\Big] \qquad (8)$$

The intensity of tangential force applied by the restraints is again $p = \dfrac{NWv}{2\pi R}$

In the foregoing expressions for the constants C, D. F, and S:

$$\Psi_1 = \frac{\sinh\gamma\pi}{\gamma}\left[\frac{\beta(\alpha^2+\beta^2-\gamma^2)\sin\alpha\pi\cosh\beta\pi + \alpha(\alpha^2+\beta^2+\gamma^2)\cos\alpha\pi\sinh\beta\pi}{\alpha\beta(\cosh 2\beta\pi - \cos 2\alpha\pi)}\right]$$

$$\Psi_2 = \frac{\sinh\gamma\pi}{\gamma}\left[\frac{\beta(\alpha^2+\beta^2-\gamma^2)\cos\alpha\pi\sinh\beta\pi - \alpha(\alpha^2+\beta^2+\gamma^2)\sin\alpha\pi\cosh\beta\pi}{\alpha\beta(\cosh 2\beta\pi - \cos 2\alpha\pi)}\right]$$

$$\Psi_3 = \sinh\gamma\pi\left[\frac{(\beta^2-\alpha^2-\gamma^2)\cos\alpha\pi\sinh\beta\pi - 2\alpha\beta\sin\alpha\pi\cosh\beta\pi}{\alpha\beta(\cosh 2\beta\pi - \cos 2\alpha\pi)}\right]$$

$$\Psi_4 = \sinh\gamma\pi\left[\frac{(\beta^2-\alpha^2-\gamma^2)\sin\alpha\pi\cosh\beta\pi + 2\alpha\beta\cos\alpha\pi\sinh\beta\pi}{\alpha\beta(\cosh 2\beta\pi - \cos 2\alpha\pi)}\right]$$

Ring-Girder: Forces Normal to the Plane of the Ring

If a force P_N acts downwards normal to the ring at $\psi = \pi$ it is again assumed that the N discrete supports are replaced by a continuous medium of the total stiffness. Thus, if a force V is required to displace the apex of one bipod vertically through unit distance the stiffness of the equivalent uniform medium is $\rho = \dfrac{NV}{2\pi R}$.

Let EI_0 be the flexural rigidity of the ring-girder against bending normal to its own plane,

CJ its torsional rigidity,

$$n = \frac{EI_0}{CJ}$$

$$K_2 = \rho R^4/EI_0$$

$$Q_1' = \frac{1}{27} + \frac{K_2(3n+2)}{6}$$

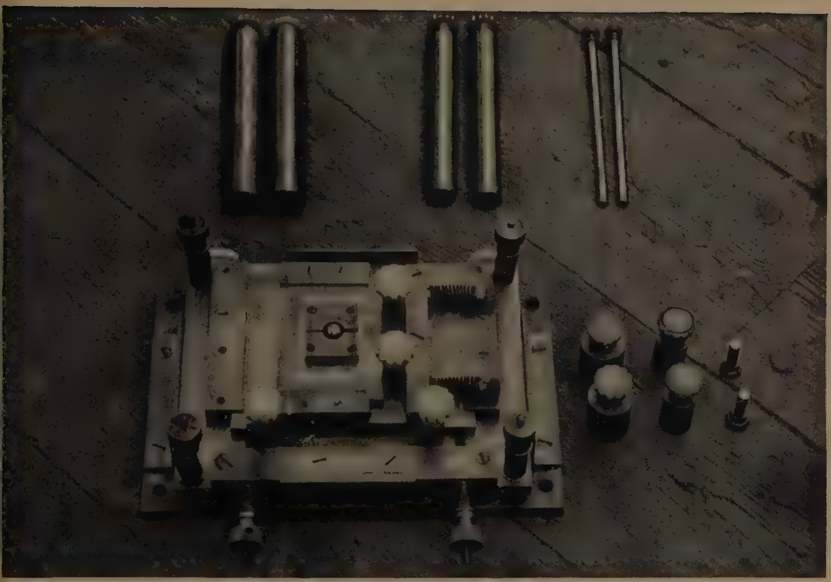
$$Q_2' = \left[\frac{K_2}{27}\left\{K_2^2 + 2K_2 + 1 + n\left(\frac{27nK_2}{4} + 9K_2 + 1\right)\right\}\right]^{\frac{1}{2}}$$

$$X_1 = (Q_1' + Q_2')^{\frac{1}{3}} + (Q_1' - Q_2')^{\frac{1}{3}}$$

$$Y_1 = (Q_1' + Q_2')^{\frac{1}{3}} - (Q_1' - Q_2')^{\frac{1}{3}}$$

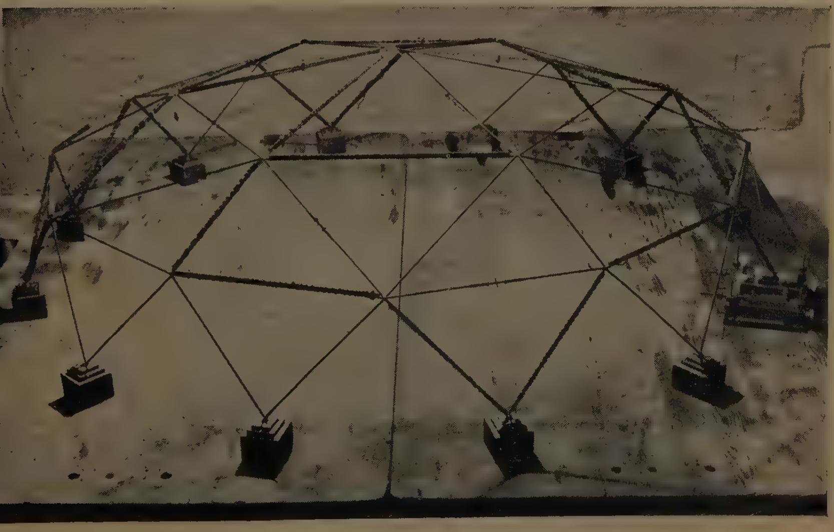
$$\alpha_1 = \frac{1}{\sqrt{2}}\left\{\left(\frac{X_1^2}{4} + \frac{2X_1}{3} + \frac{4}{9} + \frac{3Y_1^2}{4}\right)^{\frac{1}{2}} + \left(\frac{X_1}{2} + \frac{2}{3}\right)\right\}^{\frac{1}{2}}$$

Fig. 4



View of Space Deformeter

Fig. 5



Model of Simple Dome

Fig. 6

MODEL OF DOME OF DISCOVERY

Fig. 7

Model of Upper Part of Dome of Discovery: Rigid Supports in place of Ring-Girder

$$\beta_1 = \frac{1}{\sqrt{2}}\left\{\left(\frac{X_1^2}{4} + \frac{2X_1}{3} + \frac{4}{9} + \frac{3Y_1^2}{4}\right)^{\frac{1}{2}} - \left(\frac{X_1}{2} + \frac{2}{3}\right)\right\}^{\frac{1}{2}}$$

and $\gamma_1 = \left(X_1 - \frac{2}{3}\right)^{\frac{1}{2}}$

Then it can be shown [5] that the downward displacement of the ring at the point defined by ψ is :

$$w = \frac{P_N R^3}{3\sqrt{3}EI_0Q_2'} \cdot \alpha_1\beta_1\gamma_1 \operatorname{cosech} \gamma_1\pi \left[\frac{n-\gamma_1^2}{\gamma_1^2} \cosh \gamma_1 \psi \right.$$

$$+ \frac{1}{(\alpha_1^2+\beta_1^2)^2}\left\{-2\alpha_1\beta_1 n(\Psi_2 \cos \alpha_1\psi \cosh \beta_1\psi + \Psi_1 \sin \alpha_1\psi \sinh \beta_1\psi)\right.$$

$$+ \{(\alpha_1^2+\beta_1^2)^2 + n(\alpha_1^2-\beta_1^2)\}(\Psi_1 \cos \alpha_1\psi \cosh \beta_1\psi$$

$$\left.\left. - \Psi_2 \sin \alpha_1\psi \sinh \beta_1\psi)\right\}\right] \quad . \quad (9)$$

where Ψ_1 and Ψ_2 are obtained from the same expression as for loading in the plane of the rim but with α_1, β_1, and γ_1 replacing α, β, and γ.

The intensity of loading at ψ is then pw and the reaction at any support can be found by integrating pw either directly or graphically over the appropriate length of the ring-girder.

The bending moment in the ring is :

$$M' = \frac{P_N R}{3\sqrt{3}Q_2'} \alpha_1\beta_1\gamma_1 \operatorname{cosech} \gamma_1\pi \left[(\gamma_1^2+1) \cosh \gamma_1\psi \right.$$

$$+ \{2\alpha_1\beta_1\Psi_1 - (\alpha_1^2-\beta_1^2-1)\Psi_2\}\sin \alpha_1\psi \sinh \beta_1\psi$$

$$\left. + \{2\alpha_1\beta_1\Psi_2 + (\alpha_1^2-\beta_1^2-1)\Psi_1\}\cos \alpha_1\psi \cosh \beta_1\psi\right] \quad . \quad (10)$$

Torsional effects were small in the case considered and the long expression for the torque has in consequence been omitted. It will, however, be found on p. 237 of reference [4] and may be needed for cases of small radius rings.

Experimental Verification of Ring Formulae

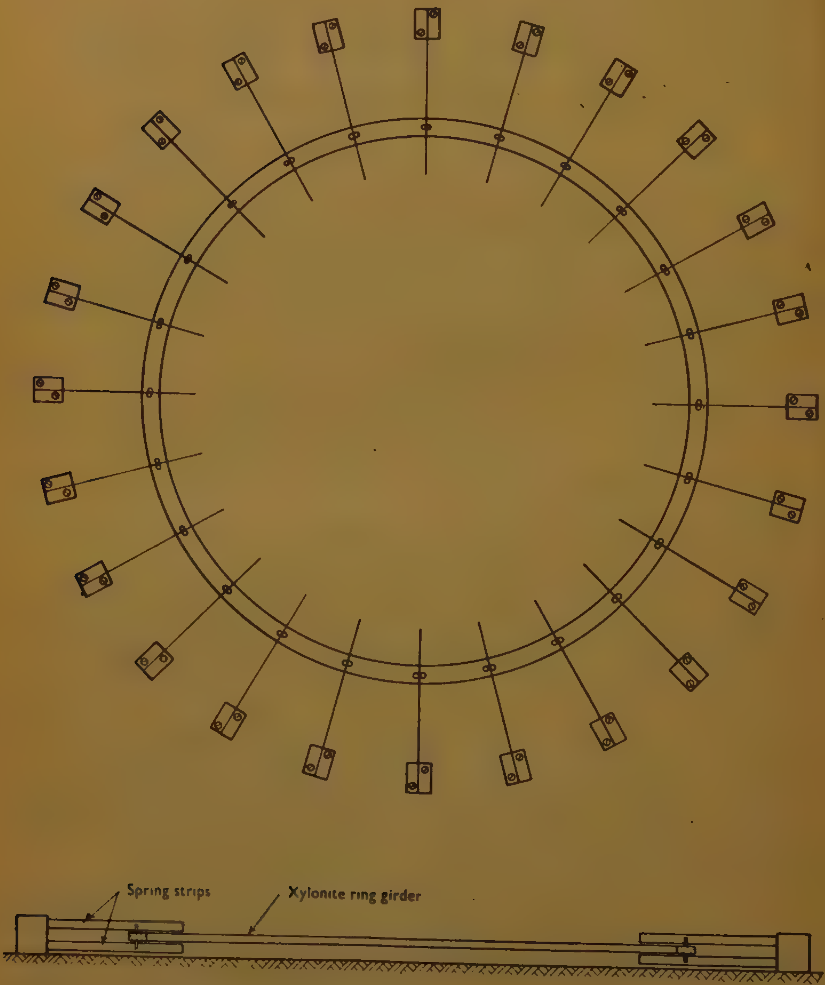
The formulae (2), (3), (6), and (7) were verified by experiments upon a ring of 18 inches diameter, $\frac{3}{8}$ inch by $\frac{3}{16}$ inch in cross-section, cut from a sheet of xylonite and mounted as shown in *Figs 9* (p. 438).

The small spring strips supporting the ring allowed free radial movement but resisted tangential movement as required by the conditions of the analysis. Radial and tangential point loads of 1 lb. were applied separately in the plane of the ring and displacements of a number of points were measured. A comparison of the figures obtained with those

calculated from the formulae is given in *Fig. 10* for the radial and in *Fig. 11* (p. 440) for the tangential load.

To verify formula (9) for loads normal to the plane a mild-steel ring was suspended from twenty-four similar and equally spaced helical springs. Equal loads were suspended from the ring at each supporting point to give the springs initial tensions. An additional point load of 1 lb. was then suspended from support 13 and the ring displacements were measured.

Figs 9

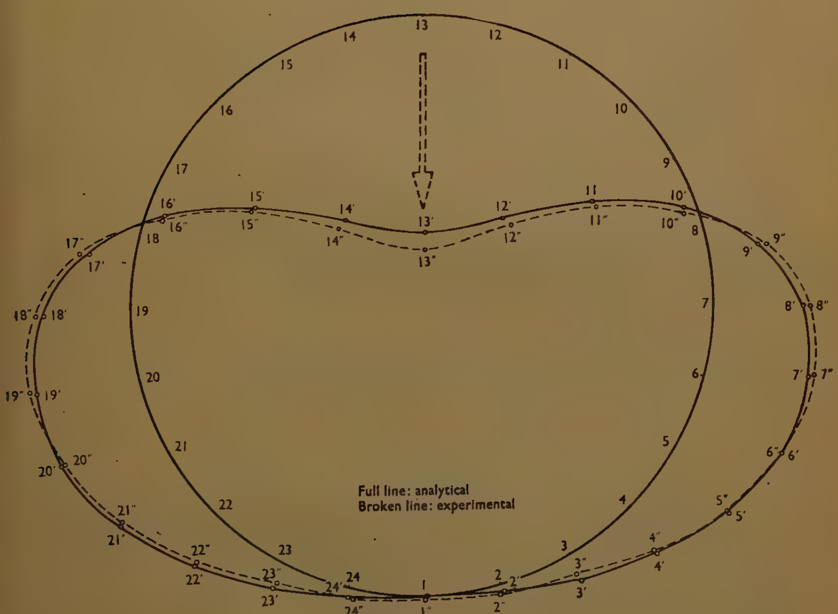


EXPERIMENTAL RING-GIRDER

The agreement between these and the calculated values is practically exact as shown in *Fig. 12* (p. 441). Alternate springs were then removed so that the ring was supported at twelve equidistant points and the experiment repeated. The result is also shown in the same Figure, the agreement between calculated and experimental values again being almost exact. Repetitions of this procedure left six and finally only three supporting points, and the differences between the measured displacements and those calculated on the assumption of continuous supports are seen from *Fig. 12* to be remarkably small.

These experiments leave little room for doubt as to the accuracy of the analysis and since all the calculated displacements show such good

Fig. 10



COMPARISON OF EXPERIMENTAL AND ANALYTICAL DISPLACEMENTS OF A RING-GIRDER RESULTING FROM A RADIAL LOAD

Fig. 11

Comparison of Experimental and Analytical Displacements of a Ring-Girder resulting from a Tangential Load

agreement with measured values it may reasonably be assumed that the calculated bending moments would be equally reliable.

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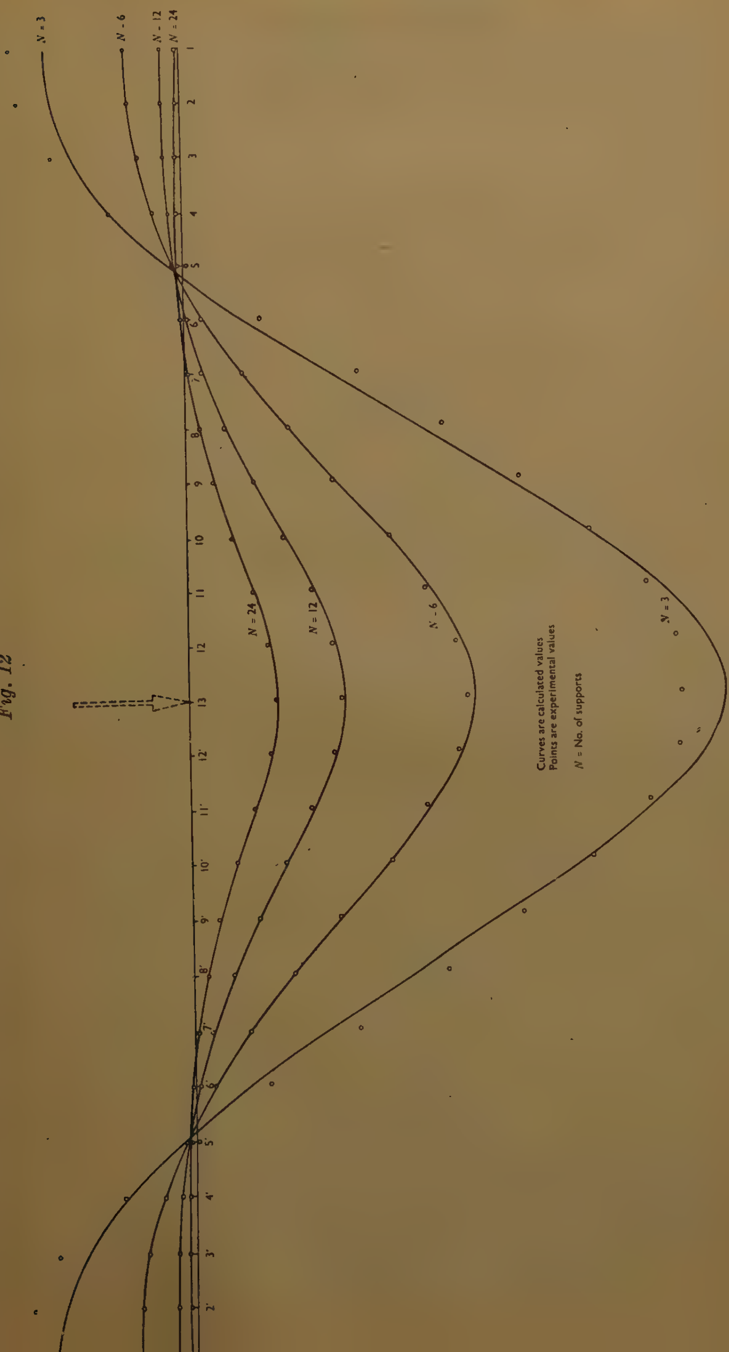
1. A. J. S. Pippard and S. R. Sparkes, "Simple Experimental Solutions of Certain Structural Design Problems." J. Instn Civ. Engrs, vol. 4, p. 79 (Nov. 1936).
2. F. Bleich, "*Stahlhochbauten*" ("Steel Structures"), vol. 2. Julius Springer, Berlin 1933.
3. T. O. Lazarides, "The Structural Analysis of the Dome of Discovery." Crosby Lockwood, London, 1952.
4. A. J. S. Pippard, "Studies in Elastic Structures." Edward Arnold & Co., pp. 19–24.
5. Ibid., pp. 236–237.

Fig. 12



DEFLEXIONS OF A RING-GIRDER RESULTING FROM A LATERAL LOAD

CORRESPONDENCE
on Papers published in Proceedings, Part III, August, 1952

Works Construction Paper No. 20

"The Construction of Loch Sloy Dam" †

by

James Stevenson, B.Sc., A.M.I.C.E.

Correspondence

Mr F. N. Sparkes observed that the strength of the concrete referred to in Table 1 was lower than was to be expected from the water/cement ratios stated to have been used. For Portland cement of average quality the actual average compressive strength of the test cubes at 7 days was about 16 per cent lower than was to be expected and at 28 days about 24 per cent lower. That was clearly not the result of faulty compaction in the test cubes since the figures showed that the density obtained was almost as high as the theoretical density when calculated from the mix proportions and the quoted specific gravities. Could the Author give any indication of the quality of the cement which was used in the works ? If tests on the cement showed that—even though complying with the British Standard—its quality was not much above the minimum requirements, then low concrete strengths were to be expected. Where large quantities of concrete were being laid, as in the case quoted by the Author it would seem to be well worth while carrying out a preliminary investigation on cement from works economically available for the site so that the highest-quality cement might be chosen for the work. Had it been possible to secure the expected average strengths from the stated water/cement ratios, leaner mixes could have been used to secure the specified strength with a resulting economy. The saving of only 1*s*. per cubic yard would have resulted in an overall economy of £10,000 in the cost o the concrete for the work referred to ; cases were known where previou selection of the source of cement had resulted in savings of up to 6*s*. pe cubic yard of concrete. If therefore the Author could throw any light o the possible reasons for the comparatively low strengths recorded it woul be of material assistance in the future to engineers engaged in the desig and construction of works employing large quantities of concrete.

The Author had not given an indication of the variability in th

† Proc. Instn Civ. Engrs, Part III, vol. 1, p. 169 (Aug. 1952).

strengths in the test cubes. Such information would be of interest in relating the variability of the concrete to the methods used to control the quality. That would also react on the economy of the work in that a greater uniformity in the strength of the test cubes would allow, in itself, a leaner mix to be used to comply with a given minimum strength requirement.

Mr P. O. Wolf felt that the Paper had not been intended to describe the design of the Loch Sloy Dam and he would not, therefore, refer to matters of design.

He asked the Author to complete his record by adding the dates of letting the contract and of completion to the other interesting dates mentioned in the Paper.

What had been the numbers of engineers and other staff, of foremen, and of skilled and unskilled labour employed at Sloy Dam at the time of peak progress? Could the Author produce a chart showing the contractor's site organization? Mr Wolf's impression had been that the site organizations of both the contractor and the engineer were small in number, but the results were a tribute to their efficiency.

Turning to some of the constructional details he observed that the hydraulics of high-velocity flow over the spillway would demand a perfectly smooth apron; to what accuracy had the spillway beams had to be set?

On p. 188 mention was made of the 2-foot-6-inch-thick upstream layer of rich concrete. He recollected that the contractor had asked to be allowed to cast the two qualities of concrete without the use of a vertical "draw-sheet" between them. It would be interesting to know the actual methods adopted and the average thickness of rich concrete which the contractor had found necessary in enabling him to place the required minimum of 2 feet 6 inches, and whether in the light of his experience he was satisfied that he had found the most economical method of constructing the continuous membrane along the upstream face.

Shuttering had been mentioned at some length in the Paper and in the discussion. The problem of the choice of the correct thickness and number of shutters was of great interest to engineers engaged on the construction of large concrete structures. The height of lift adopted at Sloy was somewhat less than the thickness of shutter chosen would have supported. It would be valuable to know if it had been found that a gain had actually resulted from that greater weight in increasing the number of uses of each panel.

He regretted that Mr Parry, who had special experience in that field, had been ill and unable to attend the meeting for he would, no doubt, have dealt with the question of welding steel shuttering. Mr Parry had himself advised the use of welded shuttering on other hydro-electric schemes, but it appeared that the manufacture of accurate multiple panels required a degree of care beyond that usually taken by a welder.

The provision by the Board of the cableways had been criticized, but they had quite clearly proved to be of constructional value at the Sloy Dam alone. Since the same cableways would be used again in the construction of other dams, there could be no doubt that their adoption would lead to considerable long-term economies as well.

The Author, in reply, noted Mr Sparks's observation that the actual average of the compressive strengths of the test cubes was lower than could be expected, but thought it unlikely that that had been caused entirely by one circumstance; there had probably been a number of contributing factors.

It was possible that, as Mr Sparks had suggested, the quality of the cement had partly caused those low strengths. The cement had not all been supplied from the same source, and there had been times when, owing to an acute shortage of cement in Britain, imported Continental cement had had to be used. Now and again the cement had only just passed the British Standard requirements.

There had been no alternative source of supply economically available to the site for the very large quantities required. Mr Sparks had, however, brought out an important point which could effect large savings on a job of that nature, where there were alternative sources of supply. By obtaining from preliminary investigations a series of curves showing relationship between water/cement ratio and crushing strength for each cement, a considerable saving might well be made possible by using a cement giving high strengths, even though that particular brand might be considerably more expensive (perhaps owing to increased haulage charges to the site) than the others.

The Author, however, did not want to blame the cement unduly, since he realized that there had been other difficulties which had no doubt contributed. For instance, rainwater would be continually changing the water content of the sand in the storage hoppers and stock-piles, thus making it extremely difficult to maintain an absolutely constant water/cement ratio, and, since the sand would not have time to dry out appreciably between the times of taking the daily moisture-content tests, such rainwater would always tend to increase the water/cement ratio until it was, of course, obvious that the mix was getting too wet, when adjustments would be carried out by decreasing the quantity of added water. The water/cement ratios given in Table 1, although reasonably accurate, should, therefore, be looked upon as being theoretical and minimum values.

Another factor was that a considerable number of the test cubes had not been cured thermostatically, and the results of those had been included in computing the average. There had been, in addition, an occasional slight falling-off in the efficiency of mixing, caused by wear on the mixer blades, before that became sufficiently serious to warrant replacement blades being fitted.

Neglecting about 3 per cent of the results, the minimum strengths

expressed as a percentage of the average for the various mixes were just slightly below 70 per cent. That indicated that good control had been achieved, despite the many difficulties mentioned.

In answer to Mr Wolf's queries, the Author stated that the tender had been accepted in January 1947. At the time of peak progress, maximum labour strength had reached 425, and had consisted of 4 senior foremen, 6 section foremen, and about 66 skilled, 80 semi-skilled, and 250 unskilled workmen. There had been an office staff of 13, and an engineering staff of 5 (3 civil, 1 mechanical, and 1 electrical) and, of course, one king-pin, the Agent.[1]

The Author doubted that the hydraulics of flow over the spillway would, in fact, demand a perfectly smooth apron as suggested by Mr Wolf, but stated, however, that normally at the joint between one beam and another the top of one beam would align almost exactly with the lower edge of the next beam, and steps of more than $\frac{1}{16}$ inch between beams had been rare. The joints between beams, however, had been almost 1 inch wide at the surface, and that had permitted small discrepancies to be smoothed out in pointing. There was thus no sudden step between one beam and the next.

At the upstream face, as Mr Wolf had stated, division plates which had originally been used to separate the front 2 feet 6 inches of rich concrete had proved unsatisfactory, and by degrees the procedure had resolved into placing additional Class 5 concrete to enable the division plates to be dispensed with. In a total of 6,400 cubic yards of concrete, the quantity of cement used in excess of that which would have been used if the theoretical demarcation line between Class 5 and Class 3 concretes had been adhered to was 3,650 cwt, which represented an excess of 0·56 cwt per cubic yard. Those figures showed that an average thickness of 3 feet 5 inches of rich concrete was necessary to guarantee the 2-foot-6-inch minimum, and the Author thought that although a saving was effected by dispensing with the division plates, there was not a great deal in it one way or the other. The Class 5/2½ concrete at the face had been a nuisance, and he thought that research could usefully be carried out to determine if that richer concrete was really so desirable.

Mr Wolf had also commented on the weight of the shutters, but it was the Author's opinion that the thickness chosen for the plate had been very near the optimum for that particular job. It had been necessary to train shutter men up from labourers, and because of that the shutters had received much rough handling. Owing to the thickness, very little repair work had been required during construction, but at the end of the job the shutters had required considerable maintenance to ensure that they were dispatched to the Contractors' Regional Plant Depot in good condition.

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[1] A copy of the site-organization chart is filed with the MS. of the Paper in the Institution Library.—SEC. I.C.E.

Structural Paper No. 30

"Some Recent Experience in Composite Pre-cast and In-situ Concrete Construction, with Particular Reference to Pre-stressing" †

by

Felix James Samuely, B.Sc.(Eng.), A.M.I.C.E.

Correspondence

Mr Thomas Ridley observed that the obvious disadvantages in the site manufacture of ferro-concrete were such as to have caused considerable concern over the years of its use, and still had not been satisfactorily eliminated. Perhaps the most important of those disadvantages were the following :—

(1) Manufacture of good concrete with consistent qualities under all weather conditions was extremely difficult.

(2) The successful handling and placing of wet concrete called for great skill and was extremely laborious, involving expensive equipment.

(3) The falsework required in general structural work had no value in the permanent works, and therefore represented unrewarded effort.

(4) The successful curing of ferro-concrete required careful attention.

(5) The organization required in supplying to site the raw materials for concrete manufacture represented a large part of its cost.

It would seem logical, therefore, to resort to in-situ ferro-concrete work only when consistently good concrete could be maintained, and large enough quantities continuously placed so as to achieve an economical proposition. Yet in structural work those conditions did not always arise, and the obvious conclusion was that the consistency and quality required for such works were best obtained from an alternative method of manufacture. The organization available in a pre-cast concrete factory was surely the answer to that problem of construction. With regard to the disadvantages in pre-cast concrete work mentioned by the Author, the following factors should also be borne in mind :—

(1) The increase in transport cost was appreciable only over long distances, and that cost was somewhat eased by the saving in special equipment on site. When the saving in time of construction, the reduction in skilled trade requirements on site, and the saving in temporary materials of construction were all

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† Proc. Instn Civ. Engrs, Part III, vol. 1, p. 222 (Aug. 1952).

assessed, the economic advantage of pre-cast concrete might be great.

(2) The continuity and monolithic effect of in-situ ferro-concrete were an inherent property of that material of construction. A structure designed for the successful use of pre-cast-concrete work throughout did not always attempt to incorporate those properties, as a matter of course.

Mr Ridley thought, therefore, that the rational approach to the full utilization of the properties of ferro-concrete construction was more easily realized when the manufacture of concrete was undertaken in ideal working conditions such as were normally found in a factory making pre-cast concrete. The use of pre-cast concrete, therefore, represented an advantage over the more usual in-situ construction. With its many advantages, and very few disadvantages, it was surely a natural development in the field of structural engineering, the full value of which was not generally realized.

Mr Ulf Bjuggren, of Stockholm, Sweden, observed that composite elements and constructions had been used for the past 10 years at the AB Betongindustri factory, and the results of laboratory tests and building experience gained there had been published in the Swedish technical press.[1, 2]

The connexion by adhesion to a concrete surface depended on the manner and quality of the work done. Solid slabs had been formed consisting of a tension zone made of prefabricated pre-stressed slabs with concrete cast in situ on top of them. Although the joint surface was smooth (having been cast against steel moulds) full adhesion had been obtained and no cracks were visible in the joint when the construction was tested to failure.[2] Some building jobs had also been carried out that way, by casting against a smooth surface,[2] but the method was not to be recommended in normal practice because of the great care which had to be taken with regard to the quality of the work. In the normal combined constructions protruding stirrups were always used, in conjunction with a roughened concrete surface. Sometimes a simple kind of indentation was used on the surface, mainly in order to satisfy the authorities.

It was correct to say that it was theoretically possible to save steel by the use of composite sections, but in practical circumstances there had to be some top-reinforcement to allow for the handling of prefabricated elements, so that there would be no great saving. A fact of greater importance was that it was usually more costly to produce an element in two operations than in one. Some years previously the AB Betongindustri

[1] U. Bjuggren, "*Den armerade betongens verkningssätt i sprickstadiet vid böjning*" ("The behaviour of reinforced concrete under bending load in the cracked stage"). *Betong*, 3/1946.

[2] U. Bjuggren, "*Sammansatta konstruktioner: strängbetong-vanlig armerad betong*" ("Composite constructions: bonded pre-stressed concrete—ordinary reinforced concrete"). *Byggnadsvärlden*, 21/1951.

had produced composite beams in order to economize in the number of standard moulds required, but it had been found in the end that beams could be produced more economically by casting them in one single operation, even if the outlay for moulds was greater.[1] At the present time, composite construction was generally used only when a slab, cast in situ, was placed on prefabricated beams. In that case it was often economical to hang the moulds from the beams. An even better system had been used in the erection of a multi-storey industrial building, using prefabricated columns and beams of pre-stressed concrete. The builder had fabricated ordinary reinforced-concrete slabs on the site, and those had been placed on the beams and had functioned both as bottom moulds for the in-situ concrete and as tension zone of the solid slab.[1] For the concrete of the in-situ slab it was always sufficient to use good "building" concrete since it was never necessary (owing to the lower stresses involved) to use the high-quality concrete required for pre-stressed elements.

When casting a slab on top of prefabricated beams in composite construction it was often economical to arrange for the continuity of the mild-steel bars over the supports. While setting, the self-weight of the beams and the slab would then be freely supported, and after hardening they would act as a continuous beam for applied loads. Moreover, the stiffness of the beam under negative bending moments (acting as non-pre-stressed concrete) was less than for positive bending moments, and from that it followed that the negative bending moments over the supports would be less than was usually calculated. With a constant depth of the beam the support bending moment would decrease by about 25 per cent, a figure which had been derived from theory and later confirmed by a laboratory test.[1] For a beam with fixed ends and an evenly distributed load $q + p$, the support moments would be:

$$M_e = -0{\cdot}0833(p + q)l^2,$$

and the span moment:

$$M_f = 0{\cdot}0417(p + q)l^2.$$

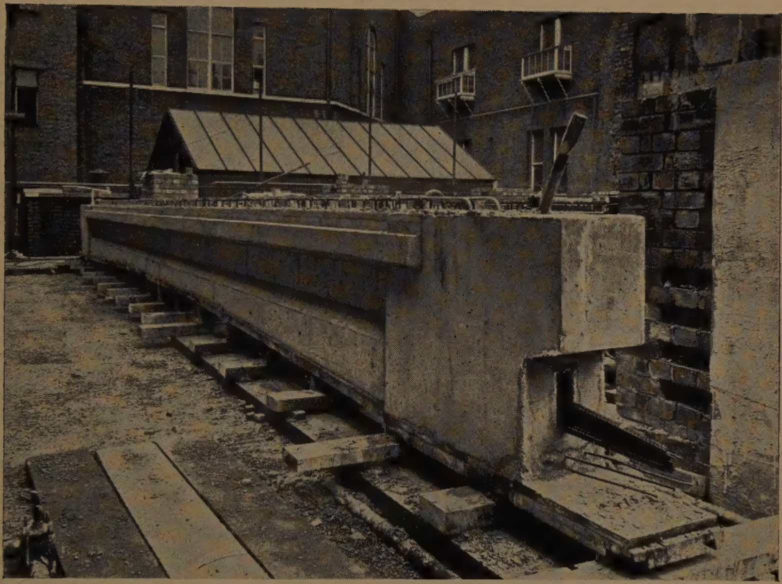
With a combined construction, in which the ends were fixed with mild steel and q denoted the weight of the beams and the slab, $M'_e = -0{\cdot}0625pl^2$ and $M'_f = 0{\cdot}0625pl^2 + 0{\cdot}125\,.\,ql^2$. In the case of $p = 2q$ it could be shown that the ratio $M'_f : M'_e$ was 2, as compared with $M_f : M_e = 0{\cdot}5$.

Mr J. W. A. Ager, noted that the Author had referred on p. 241 of the Paper to the pre-stressed-concrete beams at Queen Mary College, London. Those beams, which spanned approximately 34 feet, had been post-tensioned on the Magnel-Blaton system, prior to erection.

Fig. 58 showed one of those beams cast on the building floor immediately below the final position in the structure, where it had been prestressed before erection. The mild-steel reinforcement projecting from the

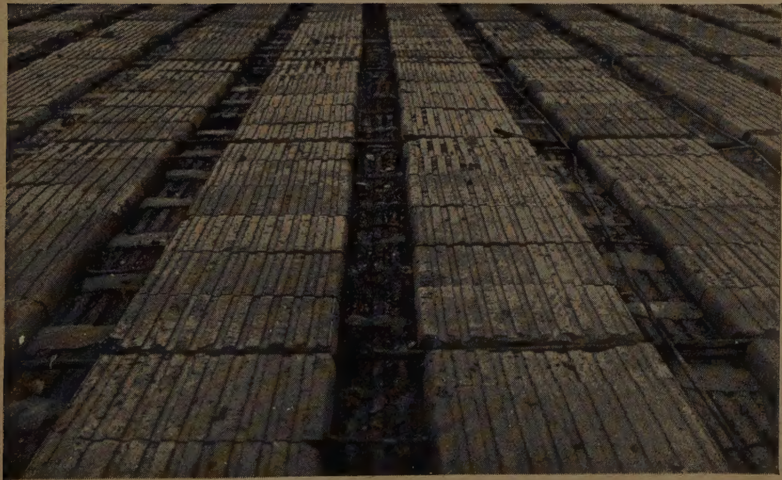
[1] See ref. 2, p. 447.

Fig. 58



34-FOOT-SPAN MAGNEL-BLATON BEAM FOR QUEEN MARY COLLEGE, PRIOR TO PRE-STRESSING AND ERECTION

Fig. 59



STAHLTON ROOF UNDER CONSTRUCTION AT THAMESHAVEN

top surface of the beam provided the shear connexion between the pre-stressed member and the in-situ slab, which had been cast as part of the finished floor.

Mr Ager wished to correct the Author's statement, also on p. 241, that Stahlton flooring " has not yet been carried out in Great Britain." In fact a number of floors of that type had been completed, and Stahlton flooring was now being produced in quantity by a British concrete firm.

Fig. 59 showed a Stahlton roof of 24 feet span in course of construction at Thameshaven.

The Author, in reply, stated that he appreciated Mr Ridley's enthusiasm for pre-cast concrete. However, whilst pre-cast concrete and in-situ concrete could exist side by side and have their own applications, he would not like to agree in principle that pre-cast concrete was, under all circumstances, an advance over in-situ construction.

The Author thanked Mr Bjuggren for his reference to the publications in the Swedish technical press. It appeared that the economic situation in Sweden was somewhat different from that in Britain, where 1 cubic foot of pre-stressed concrete, including moulding, erection, etc., cost about 5 to 6 times the amount of 1 cubic foot of in-situ concrete ; furthermore, since no special shuttering was required for the topping and the amount of steel required was a minimum, it was considerably cheaper to have as much reinforced concrete cast in situ as possible, provided that the soffit was pre-cast and pre-stressed. It might be found that the situation was different in the case of large buildings where pre-casting and post-tensioning could be carried out at a site at a cheaper rate than in the workshop.

The Author apologized to Mr Ager for saying that the Stahlton flooring had not yet been carried out in Great Britain. That statement had been correct at the time the Paper was being prepared and the Author was fully aware that Stahlton flooring had since been constructed in Great Britain.

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